Crisis Bargaining Over What?

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Abstract

Uncertainty about power and resolve has dominated the literature on information problems in crisis bargaining. Using a game-free approach, I study the neglected case of uncertainty about a common value of the contested object, or *stakes*. Whether always-peaceful equilibria exist depends on whether states bargain over shares or transfers: allocating shares can facilitate always-peace, but war occurs with positive probability in every equilibrium of any crisis bargaining game that requires trading the object for transfers, such as side payments or concessions on other issues. I also consider the general case where states may bargain over both shares and transfers. Under mild conditions, there is no crisis bargaining game in which transfers can remedy a bargaining failure caused by an indivisibility. Correlated information can exacerbate the problem by amplifying demands when stakes are thought to be high.

Keywords: conflict, bargaining, information, game theory, mechanism design

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1 Introduction

Why do states resort to costly war when a peaceful settlement could make each better off? This question is at the heart of international politics, and one of the more compelling explanations is private information. As Fearon (1995, p. 381) maintained, "rational leaders may be unable to locate a mutually preferable negotiated settlement due to private information about relative capabilities or resolve and incentives to misrepresent such information." This insight has led to a vast body of work on information problems and war (Ramsay, 2017).

There is, however, a variety of uncertainty that the crisis bargaining literature has overlooked: uncertainty about a common value of the contested object, or the *stakes* of the dispute. This is a major oversight, as this type of uncertainty is ubiquitous in international conflict. Indeed, in any dispute, both sides are unlikely to have exactly the same information about the value of what they are fighting over. Even when, for example, oil reserves on disputed land have been precisely measured and extraction costs are well-understood, the territory's real value depends on qualities of an uncertain future. Profits depend on market prices that reflect global supply and demand, which are in turn dependent on trading relations that evolve alongside geopolitical partnerships. States must form expectations about this value, which they do from different informational vantage points. Private information about stakes, like that about power and resolve, can generate an incentive to misrepresent that information, opening the door to bargaining failure and war.

In this paper, I present an analysis of crisis bargaining games in which states possess private information about the stakes of the dispute. The focus is on war and peace as possible equilibrium outcomes while remaining agnostic about the structure of the game, such as actions and timing. This game-free approach employs tools from Bayesian mechanism design that have proven useful for studying information problems in war due to their ability to facilitate general statements about broad classes of games (Banks, 1990; Fey and Ramsay, 2009, 2011). Accordingly, the results established in the following analysis apply to every crisis bargaining game and do not hinge on the assumptions of a particular setting.

The crisis bargaining game form that I study extends an existing class from the literature, as defined by Fey and Ramsay (2011). In addition to allocating shares of the object, I allow for the possibility that the terms of a peaceful agreement include an exchange of transfers, such as side payments or compromises on other issues. When the stakes of the dispute are commonly known, shares and transfers are equivalent: a half share of the object yields the same payoffs to both states as a trade of the object for a transfer of one half its value. With uncertainty about stakes, however, it is worthwhile to incorporate the distinction, as the nature of the bargaining protocol has important implications on incentives and the consequent prospects of peace.

To demonstrate, I separately examine and compare two extreme subclasses of crisis bargaining games: the *share-protocol* subclass, defined as those in which states may bargain only over share and may not exchange transfers, and the *transfer-protocol* subclass, defined as those in which states may not bargain over share but may agree to a peaceful trade of the object for transfers. Always-peaceful equilibria can exist in share-protocol games; for example, by agreeing to divide the object according to the known balance of power (Proposition 1). In contrast, provided the costs of war are not too large, there does not exist a transfer-protocol crisis bargaining game that admits an always-peaceful equilibrium (Proposition 2). This is because when bargaining is constrained to transfers, states face a fundamental tension: if one state offers a large transfer to compensate an opponent that claims the stakes are high, then an opponent with low expectations has an incentive to make the same demand. Anticipating this, the extending state will not concede transfers large enough to always satisfy states with high expectations. No transfer scheme can resolve this tension and, as a result, war will occur with positive probability.

I then turn to the general case where states may bargain over both shares and transfers. Of course, because always-peace is possible with shares alone, it is also possible in general. More interestingly, I consider whether transfers can help achieve always-peace when it is otherwise impossible to settle with shares alone. I refer to any such constraint as an *indivisibility*.[1] An indivisibility has widely been viewed as a surmountable cause of war: transfers in the form of side payments, linkages, or concessions on other issues can be used to compensate a state for accepting a less-than-satisfactory share, rendering any disputed object effectively divisible (Fearon, 1995, p. 389). Jackson and Morelli (2007, p. 1356) state the claim plainly: "if one allows for bargaining and transfers, war should not be possible." This logic has been regularly argued in the literature, dating back to at least Morrow (1986) and Morgan (1990).

This conventional wisdom does not hold when there is uncertainty about the stakes. Theorem 1 establishes the impossibility result for one-sided indivisibilities, i.e., situations in which the feasible set of peaceful settlements excludes shares to one side's extreme. When settlements with "fair" shares that would guarantee peace are no longer possible, states must rely on transfers to bridge the gap between the settlements with feasible shares and their privately held expectations about the object's value. Then, the same tension that plagues transfer-protocol games emerges: transfers large enough to compensate the side making concessions on their share allocation simultaneously invite extreme demands by states with low expectations. At the same time, transfers that deter mimicry from states with low expectations fail to satisfy those with high expectations. As a result, there does not exist an equilibrium of any crisis bargaining game in which transfers remedy a bargaining failure caused by a one-sided indivisibility.

I also consider two-sided indivisibilities, where infeasible shares belong to an interior portion of the bargaining space. Because feasible shares exist on each side of the indivisibility, always-peace can be facilitated by randomizing over extreme feasible settlements so that each type expects a favorable outcome on average (Proposition 3), a mechanism briefly conjectured by Fearon (1995, p. 389) and elaborated on by Powell (2006) and Wagner (2010). However, such a mechanism would not only require states to reach a peaceful agreement but to maintain it after the terms are revealed. In the anarchic environment of crisis bargaining, there is no external enforcement to hold states to unfavorable realized terms, and a dissatisfied state can always abandon the agreement and fight. Under such robustness conditions, war occurs with positive probability in every equilibrium despite

[1] An indivisibility, as defined here, can stem from any source. For example, it may arise from characteristics of the disputed object, institutional limitations of the resolution process, or other features of the strategic interaction.

the possibility of transfers. Theorem 2 presents the corresponding impossibility result.

Finally, I show that this problem can be exacerbated when each side's private information is correlated. Although correlation introduces a *screening* effect by allowing states to partially infer their opponent's type, it also creates an *amplifying* effect: when a state receives an extreme signal, it forms more extreme expectations about the stakes and bargains more aggressively. This is especially true if there are complementarities in states' signals, meaning that a marginal increase in one state's signal raises the stakes more when their opponent's signal is high. Proposition 5 presents an impossibility threshold under correlated information, with Proposition 4 demonstrating that the amplifying effect can outweigh the screening effect, making always-peace more difficult to achieve as information becomes more correlated.

The paper makes theoretical contributions to the literature on international conflict. First, I introduce uncertainty about the stakes of the dispute to the crisis bargaining framework and use a game-free approach to investigate what this form of uncertainty implies for every crisis bargaining game. This builds on a tradition that originates with Banks (1990) and most closely resembles Fey and Ramsay's (2009; 2011) study of two-sided uncertainty about power and resolve. Private information about power and resolve has dominated the literature on information problems and war since Morrow (1989), and is commonly referred to as the "two standard sources" of uncertainty (Spaniel, 2023). Uncertainty about the dispute's stakes leads to fundamentally different incentive problems than power and resolve, with new strategic implications.

Second, the results reveal how the structure of dispute resolution processes can systematically affect the prospect for peace. I establish a substantial difference in the ability to consistently achieve peace between settings where states bargain over shares and those where they bargain over transfers. Further, I show that the long-standing presumption that transfers can overcome bargaining indivisibilities does not survive the introduction of uncertain stakes. This contributes to a classic literature in international politics on side payments and issue linkage, from Stein's (1980) foundational treatment of linkage as a means of expanding the scope for settlement, to later work that argues for qualifications to the effectiveness of linkage strategies (Hassner, 2003; Goddard, 2006).

Third, the extension on correlated information qualifies the standard intuition that reducing uncertainty unambiguously promotes peace (Meirowitz et al., 2022; Sartori, 2002; Kurizaki, 2007; Ramsay, 2011). Identifying competing screening and amplification effects of correlated information, and showing how always-peace can become more difficult as a result, highlights that the relationship between information and war is not necessarily straightforward.

Finally, the paper makes a methodological contribution by presenting a new characterization for the crisis bargaining game form, which allows for an exchange of transfers alongside an allocation of shares in peaceful settlements. The framework also accommodates mixed strategies and stochastic settlements, which the previous formulation in Fey and Ramsay (2011) does not admit. The paper thus advances a growing agenda that employs mechanism design to study features of crisis bargaining settings, including ultimatums (Fey and Kenkel, 2021), intermediate policy responses (Kenkel and Schram, 2025), leader political bias (Liu, 2022), mediation (Hörner, Morelli and Squintani, 2015; Meirowitz et al., 2019), and multilateral treaties (Morrow and Cope, 2021).

2 Information and incentives in conflict

Before proceeding to the analysis, it is worthwhile to briefly discuss the problem of uncertain stakes in the context of previously studied information problems in crisis bargaining. The primitives of a crisis bargaining model include

1. the value of the object in contention v,
2. power or the probability of victory in war p, and
3. resolve or the costs of war c.

If war occurs, each state i receives the object of value v_i with probability p_i at cost c_i, where each primitive may be specific to i, usually with the constraints that all are strictly positive and probabilities sum to 1. The quantity $w_i = v_i p_i - c_i$ is thus referred to as the war payoff of the crisis bargaining game. When at least one state is uncertain about p, there is uncertainty about power. When at least one state i is uncertain about their opponent j's privately known cost-to-value ratio c_j/v_j, there is uncertainty about resolve. Due to this connection between the object's value and the costs of war, it has become standard to normalize the value to 1 for all states and focus on settings where states have uncertainty about each other's costs (Spaniel, 2023, p. 15).

My analysis departs from this standard, instead considering uncertainty about the stakes of the dispute, which includes not only uncertainty about an opponent j's value for the object v_j but also their own value v_i. To my knowledge, no previous work in the crisis bargaining literature has allowed for states to know the power balance and the costs of war but not their own value for the object. Because allowing for jointly unknown private values is equivalent to introducing dual uncertainty about stakes and resolve,[2] I choose to focus specifically on an unknown *common* value $v_i = v_j = v$ to isolate the effect of uncertainty about the stakes.

This is especially consequential when we introduce the opportunity to settle with both share allocations and external transfers. Under standard approaches, the terms of a peaceful settlement are denominated in units of the contested object's value by default. When the value is known to be 1, receiving an x share of the object is the same as receiving a transfer worth an x share, which is the same as receiving the full object in exchange for a transfer worth a $1 - x$ share. However, when states do not know the value of the contested object, no such equivalence can be established, as transfers cannot be denominated in units of an unknown quantity.

Uncertainty about the stakes of the dispute is therefore structurally different than these other sources. Table 1 summarizes the distinction between the two standard sources of uncertainty and uncertainty about stakes. In the standard cases, a change in a state's private information shifts the relative attractiveness of war as an outside option, but does not alter the utility conferred by any given peaceful settlement. In contrast, with uncertainty about stakes, increasing a state's expectation

[2] To see this, note that any pair of positive valuations (v_i, v_j) can be rewritten in terms of a common scale component and a separate resolve component. Then, normalizing by the resolve component in the typical way returns us to an equivalent representation with common value v and effective costs of war that reflect resolve.

Uncertainty	War payoff	Peace payoff	Terms of peace	Incentive
Power	Unknown, incr. in p_i	Known, constant	Share $\sim$ transfer	Overstate p_i
Resolve	Privately known, decr. in c_i	Known, constant	Share $\sim$ transfer	Understate c_i
Stakes	Unknown, incr. in v	Unknown, incr. in v	Share $\not\sim$ transfer	Ambiguous

Table 1: Comparison of information problems in crisis bargaining. *Note:* In the war payoff and peace payoff columns, the first element indicates whether the payoff is known to all states, privately known to state i, or unknown to all states (given the terms of settlement). The second element after the comma indicates how the payoff changes in the uncertain parameter. The terms of peace column indicates whether shares and transfers are commensurable ($\sim$) as settlement instruments.

for the object's value results in both a greater return to fighting as well as an increased value for reaching peace at given settlement terms. Both the war payoff and the peace payoff to move in tandem with changes to a state's private information.

These structural differences produce different incentives. When there is uncertainty about power and resolve, states are always strategically motivated to overstate their strength and resolve (i.e., understate their costs). With uncertainty about stakes, however, the incentive to misrepresent is not straightforward and varies setting to setting. For example, in a standard ultimatum crisis bargaining game over shares, the offer-receiving state takes into account the expected stakes when choosing whether to accept or reject. In this situation, the receiving state will make greater concessions the less they expect the stakes to be, for the same reason less resolved states are more willing to make greater concessions. Knowing this, the proposer would have incentive to overstate v if the receiver were to take them at their word. At the same time, the receiver would have incentive to understate v if the proposer were to take them at their word.

Several recent models study sources of uncertainty that do not fit into these categories. For example, in Dong (2025), the source of a bargaining delay is uncertain. Bils and Spaniel (2017) consider a dispute is over a policy decision where each side has uncertainty about each other's spatial preferences. In another recent article, Spaniel and Bils (2018) study uncertainty over a state's "moderation," operationalized as the share of the object they allow their opponent to keep after victory. This latter paper has the interesting quality of being conceptually similar to resolve uncertainty while being technically equivalent to power uncertainty.

3 Crisis bargaining games

In this section, I define the class of crisis bargaining games. The definition mostly follows the crisis bargaining game form defined by Fey and Ramsay (2011), but is made slightly more general to better accommodate the analysis of our information problem of interest.

A crisis bargaining game Γ is an interaction between two states $i = 0, 1$. Throughout the paper, I use j to denote a state i's opponent. Each state chooses actions $a = (a_0, a_1) \in A \equiv A_0 \times A_1$, which lead to a final outcome of war or peaceful settlement. In war, each state incurs costs $c_i > 0$ to fight for an object they value $v_i > 0$, which they win with probability $p_i = 1 - p_j$. Both probabilities are pinned down by $p = p_0$. Therefore, we can denote each state's war payoff by $w_i = v_i p_i - c_i$. On the other hand, peaceful settlements can include two components. First, states may agree to a share or division of the object, $x_i(a) \in [0, 1]$. For a state i to receive a share implies their opponent receives the remainder, hence $x_i(a) = 1 - x_j(a)$ for all $a \in A$. Second, states may extend and receive transfers in the form of side payments or compromises on other issues, netting to a gain of $t_i(a) \in \mathbb{R}$. For a state i to extend a transfer implies their opponent j receives it, hence $t_i(a) + t_j(a) = 0$ for all $a \in A$. From this, state i's payoff from cooperating under action profile a can be expressed by $v_i x_i(a) + t_i(a)$. Figure 1 presents several examples.

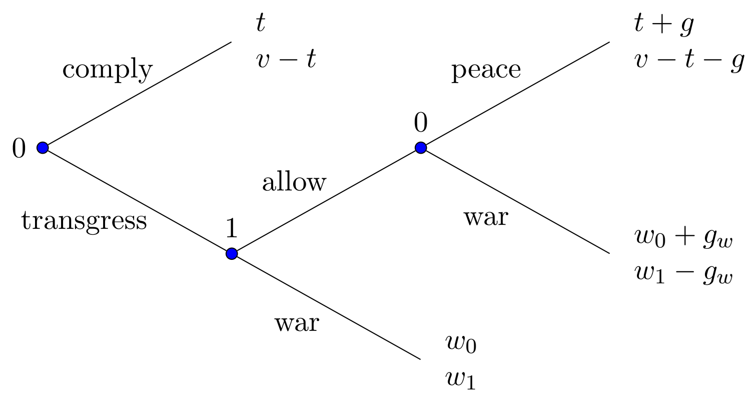
Note the following two extensions from the crisis bargaining game form as defined in previous work. First, the value of the disputed object is typically normalized to 1; however, here this value is denoted by parameter v_i for each state i, which will be treated as common and unknown in the main analysis. Second, our definition of crisis bargaining games differentiates between settlements that allocate shares from those that involve transfers. When the value of the object is normalized to 1, there is no difference between an agreement on shares and an agreement on transfers. However, as we will see, the distinction will be consequential when the value of the object is unknown.

A crisis bargaining game Γ concludes with an outcome that can be represented by $(\pi, x, t) \in \Omega \equiv \{0, 1\} \times [0, 1] \times \mathbb{R}$, where π indicates whether war occurs and, if war does not occur, x yields the share allocated to state 0 while t yields the transfer allocated to state 0. Then, we can say that any crisis bargaining game has a corresponding outcome function $\gamma : A \to \Delta(\Omega)$ that takes actions and yields a distribution over outcomes. For any action profile, the induced moments are given by $\pi^\gamma(a) \equiv \mathrm{E}_{\gamma(a)}\pi$ as the probability of war and, in a peaceful settlement, $x^\gamma(a) \equiv \mathrm{E}_{\gamma(a)}[x \mid \pi = 0]$ is the expected share of object allocated to state 0 while $t^\gamma(a) \equiv \mathrm{E}_{\gamma(a)}[t \mid \pi = 0]$ is the expected transfer received by 0. Then, the expected utility for a state i under an action profile a is

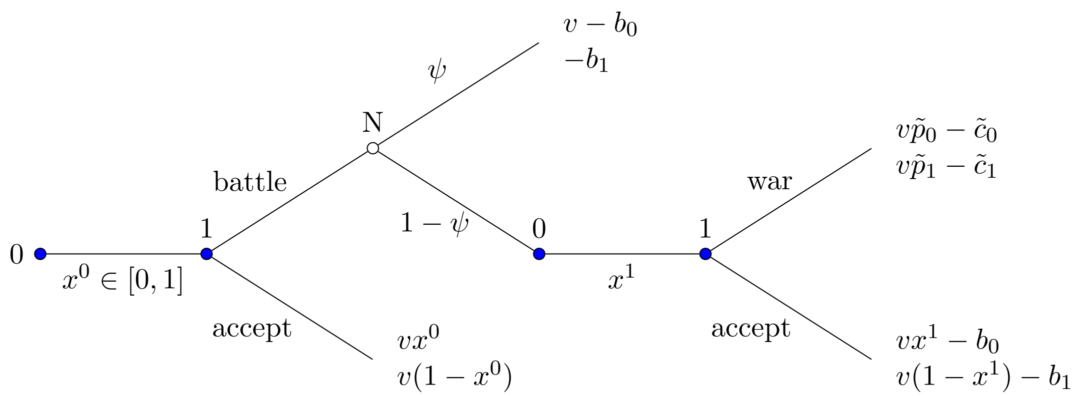
$$u_i(a) = \pi^\gamma(a) w_i + (1 - \pi^\gamma(a))(v_i x_i^\gamma(a) + t_i^\gamma(a))$$

where $x^\gamma(a) = x_0^\gamma(a) = 1 - x_1^\gamma(a)$ and $t^\gamma(a) = t_0^\gamma(a) = -t_1^\gamma(a)$.

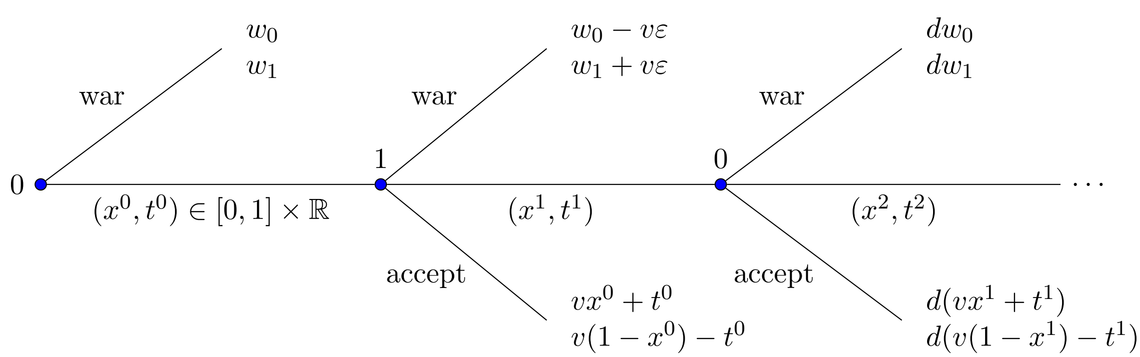
It is now time to incorporate private information. Let each state have a private type $\theta_i \in \Theta_i$ with $\Theta_i \subset \mathbb{R}$ compact and $|\Theta_i| \geq 2$. A type pair $\theta = (\theta_0, \theta_1)$ is drawn from common joint prior F on $\Theta \equiv \Theta_0 \times \Theta_1$. Private information may concern any collection of the primitives (v_0, v_1, p, c_0, c_1), though subsequent sections will focus on the case where p, c_0, and c_1 are common knowledge. When a primitive depends on types, we can write i's expected utility given θ as $u_i(a, \theta)$ to make this dependence explicit, with war payoffs likewise expressed as $w_i(\theta)$. Letting a strategy profile be defined as a function mapping types into actions, $\sigma : \Theta \to \Delta(A)$, the interim expected utility for a



(a) Gurantz and Hirsch (2017), with $g, g_w > 0$ as the gains from transgression.



(b) Smith and Spaniel (2019), with $b_i \in (0, c_i)$ as battle costs and ψ as the probability 0 wins decisively after battle. Note that $\tilde{p}_1 = 1 - \tilde{p}_0 = p_1/(1-\psi)$ and $\tilde{c}_i = (c_i - \psi b_i)/(1-\psi)$ establishes that state i's expected payoff from battle given subsequent war is w_i.



(c) Powell (2006), with $d \in [0, 1]$ as the discount factor and $2\varepsilon \in (0, 2\min_i p_i)$ as the first-strike advantage.

Figure 1: Examples of crisis bargaining games. *Note:* The game trees depict games that are similar in spirit to those in the corresponding articles, but may not be exactly the same. The notation, terminology, and payoff specifications have been slightly altered from the original versions to establish clear connections with the present analysis (e.g., Powell (2006) did not originally feature transfers). The superscripts on proposals index periods.

state of type θ_i given a strategy profile σ can then be written as

$$U_i^\sigma(\theta_i) = \int_{\Theta_j} \mathrm{E}_{\sigma(\theta_i,z)} u_i(a,(\theta_i,z))dF(z \,|\, \theta_i).$$

The only assumption we require on the action space is that it satisfies what is known as the voluntary agreements (VA) criterion. Due to the anarchic nature of crisis bargaining, it is standard to focus on settings where states can always unilaterally guarantee themselves an expected payoff of at least their war payoff. I thus make the following assumption.

Assumption 1 (Voluntary Agreements)**.** *For any crisis bargaining game* Γ*, there exists an action* $\tilde{a}_i \in A_i$ *for each state* i *such that, for all* $a_j \in A_j$ *and* $\theta_i \in \Theta_i$*,*

$$\mathrm{E}_{\theta_j}[u_i((\tilde{a}_i, a_j), \theta) \mid \theta_i] \geq \mathrm{E}_{\theta_j}[w_i(\theta) \mid \theta_i]. \tag{VA}$$

Assumption 1 is weak in that it does not assume a state's realized peace payoff exceed their war payoff to achieve peace, but only that states have actions available to them that, if taken, would guarantee at least their war payoff. Most commonly, each state can start a war. It then remains possible that a state receives less in peace than what they would expect in war *had they known* the truth. In principle, these events could lead to a delayed war or a renegotiated settlement as more information is later revealed. A stronger "ex-post" notion of the VA criterion would make always-peace more difficult to achieve.

Finally, a direct mechanism $\delta : \Theta \to \Delta(\Omega)$ is a function mapping type profiles to distributions of outcomes. Fix a Bayesian Nash equilibrium σ^* of a crisis bargaining game Γ, then δ is called an *equivalent direct mechanism* if it yields the same distribution of outcomes induced by σ^* and outcome function γ. A simple way to think about this is that there is a new game in which states only report a type to a mechanism designer, who then selects an outcome those types would have obtained in the original game. Then, for any reported type profile θ, the corresponding moments are given by $\pi^\delta(\theta) = \mathrm{E}_{\delta(\theta)}\pi$ for the probability of war and $(x^\delta, t^\delta)(\theta) = \mathrm{E}_{\delta(\theta)}[(x,t) \,|\, \pi = 0]$ for the expected share and transfer in peace. The interim expected utility of a state with type θ_i reporting as type $\tilde{\theta}_i$ under strategies σ^* can now be expressed in terms of the equivalent direct mechanism by

$$U_i^\delta(\tilde{\theta}_i;\theta_i) = \int_{\Theta_j} u_i^\delta(\tilde{\theta}_i;(\theta_i,z))dF(z \,|\, \theta_i), \text{ where}$$
$$u_i^\delta(\tilde{\theta}_i;(\theta_i,\theta_j)) = \pi^\delta(\tilde{\theta}_i,\theta_j)w_i(\theta_i,\theta_j) + (1-\pi^\delta(\tilde{\theta}_i,\theta_j))(v_i(\theta_i,\theta_j)x_i^\delta(\tilde{\theta}_i,\theta_j) + t_i^\delta(\tilde{\theta}_i,\theta_j))$$

where $x^\delta(\theta) = x_0^\delta(\theta) = 1 - x_1^\delta(\theta)$ and $t^\delta(\theta) = t_0^\delta(\theta) = -t_1^\delta(\theta)$ likewise always holds.

By definition of an equivalent direct mechanism, truth-telling in the direct mechanism yields equivalent expected utility as equilibrium play in the original game, i.e., for all θ_i, $U_i^\delta(\theta_i;\theta_i) = U_i^{\sigma^*}(\theta_i)$. Now, we have the machinery necessary to close this section with brief remarks on the mechanism design result that makes the remainder of the analysis possible.

Definition 1 (Incentive Compatibility)**.** *A direct mechanism δ is incentive compatible if and only if*

$$U_i^\delta(\theta_i;\theta_i) \geq U_i^\delta(\tilde{\theta}_i;\theta_i) \tag{IC}$$

for each state i, for all $\theta_i, \tilde{\theta}_i \in \Theta_i$.

Remark 1 (Myerson (1979))**.** *If σ^* is a Bayesian Nash equilibrium of a crisis bargaining game Γ, there exists an incentive-compatible direct mechanism δ that induces the same distribution over outcomes as outcome function γ and equilibrium strategies $\sigma^*(\theta)$.*

This result, known as the revelation principle, is exceptional in that it facilitates the analysis of broad classes of games through an investigation of direct mechanisms. In particular, because there must exist an incentive-compatible direct mechanism that produces the same outcomes for any equilibrium of a game in this class, we can make positive statements about properties of equilibria based on the outcomes that can be produced by incentive-compatible direct mechanisms.

The converse also holds but requires the condition $U_i^\delta(\theta_i;\theta_i) \geq \mathrm{E}_{\theta_j}[w_i(\theta) \,|\, \theta_i]$ for each state i and every type θ_i, which captures the equilibrium implications of the VA criterion on the direct mechanism δ (Assumption 1; see Appendix A.1, Lemma 1). Accordingly, to establish the existence of a crisis bargaining game that induces a particular distribution over outcomes, it suffices to exhibit such an incentive-compatible direct mechanism that induces that same distribution, and vice versa. In this paper, as in previous game-free analyses of crisis bargaining, I focus on the existence of games that admit always-peaceful equilibria.

Definition 2 (Always-Peaceful Equilibrium)**.** *An equilibrium σ^* of a crisis bargaining game Γ is always peaceful if and only if $\pi = 0$ for all $(\pi, x, t) \in \mathrm{supp}(\gamma(a))$, $a \in \mathrm{supp}(\sigma^*(\theta))$, and $\theta \in \Theta$.*

Remark 2. *A crisis bargaining game admits an always-peaceful equilibrium if and only if there exists an incentive-compatible direct mechanism δ such that $\pi = 0$ for all $(\pi, x, t) \in \mathrm{supp}(\delta(\theta))$ and all $\theta \in \Theta$, and $U_i^\delta(\theta_i;\theta_i) \geq \mathrm{E}_{\theta_j}[w_i(\theta) \,|\, \theta_i]$ for each state i and all $\theta_i \in \Theta_i$.*

Therefore, to recover such an incentive-compatible direct mechanism that produces an always peaceful outcome is equivalent to recovering a crisis bargaining game that admits an equilibrium that is always peaceful.

4 Uncertainty about stakes

The results from the previous section hold for any type of uncertainty about model primitives, including power and resolve. In this section, I develop additional infrastructure necessary to study the information problem of interest: uncertain stakes.

Let the probability of victory p and the costs of war c_i be common knowledge. On the other hand, both states have private information about the common value of a contested object, given by a Borel measurable function $v : \Theta \to \mathbb{R}$. I assume that v is continuously differentiable and weakly increasing in both of its arguments (without loss of generality), and that the object is always worth enough to justify fighting over it, i.e., $\min_\theta v(\theta) \geq \max_i c_i/p_i$. This latter condition is sufficient to avoid uninteresting cases where peace prevails due to expectations that the object is not valuable enough to dispute over, falling outside of the purview of crisis bargaining.

As stated in the previous section, a type pair θ is drawn from a common joint prior F on Θ. For a state i with type θ_i reporting as type $\tilde{\theta}_i$ under an equivalent direct mechanism δ, I denote the interim expected stakes by

$$V_i(\theta_i) = \int_{\Theta_j} v(\theta_i, z) dF(z \mid \theta_i).$$

Letting $\bar{\theta}_i \equiv \max \Theta_i$ and $\underline{\theta}_i \equiv \min \Theta_i$ for each state i, we can then express the maximum and minimum interim expected stakes by $\bar{V}_i = V_i(\bar{\theta}_i)$ and $\underline{V}_i = V_i(\underline{\theta}_i)$, respectively.

Of course, additional information about an opponent's type can usually be gleaned through the sequence of play. For example, a settlement offer is often observed before it is accepted, and the selected offer can reveal information about the opponent. Such posterior beliefs then serve as the basis for a state's ultimate decision to cooperate or fight.

To see this, let $\mu_i(\theta_i)$ denote the Bayesian posterior belief of state i with type θ_i after information is revealed along the path of play. Suppose we were to impose a stronger notion of the VA criterion such that states can guarantee at least their war payoff upon learning this information. Assumption 1 and Remarks 1–2 then would imply that, for any always-peaceful equilibrium σ^* of any crisis bargaining game, an incentive-compatible equivalent direct mechanism δ must satisfy

$$\mathrm{E}_{\theta_j}\left[v(\theta_i, \theta_j) x_i^\delta(\theta_i, \theta_j) + t_i^\delta(\theta_i, \theta_j) - w_i(\theta_i, \theta_j) \mid \mu_i(\theta_i)\right] \geq 0 \tag{1}$$

for all $\theta \in \Theta$, for each state i.

Inequality (1) reflects a participation constraint for peaceful settlement that depends on game-specific posteriors. Different crisis bargaining games will present different opportunities to learn information. However, regardless of how and what states learn throughout the game, all crisis bargaining games are connected by a fact: if an always-peaceful equilibrium exists, then peace must be preferred to war at all posterior beliefs that are realized on the path of play. Then, if peace is preferred to war at every such realization, it must also be preferred on average for every type.

For this reason, we do not need to deal directly with posterior beliefs that emerge from specific games: Lemma 2 (see Appendix A.2) shows that an incentive-compatible direct mechanism that satisfies inequality (1) and never produces war must also satisfy the necessary interim participation constraint $U_i^\delta(\theta_i; \theta_i) \geq V_i(\theta_i)p_i - c_i$ for each state i and all $\theta_i \in \Theta_i$. Therefore, to consider the existence of always-peaceful equilibria, we can simply consider this necessary condition for always-peace that depends exclusively on each state's interim expected utilities. If there does not exist an incentive-compatible direct mechanism that satisfies the interim participation constraint for all types of each state, there does not exist an always-peaceful equilibrium to any crisis bargaining game with uncertain stakes.

Finally, it is worth noting that the formulation developed here is highly permissive for achieving always-peace in the following sense. First, the VA criterion I employ is an interim condition, requiring only that each state expects peace to weakly dominate war *on average* upon learning their type. Second, I allow for stochastic mechanisms in which the final settlement is drawn from a distribution over outcomes, so that peace can be sustained by averaging across different settlements that may not be acceptable on their own. Both features expand the class of mechanisms available to the mechanism designer in attempting to achieve always-peace, going beyond the deterministic, ex-post framework typically employed in game-free analyses of crisis bargaining. The impossibility results derived throughout the paper, with the exception of the impossibility result for two-sided indivisibilities (Theorem 2), hold despite this flexibility.

5 Bargaining over shares or transfers

In the rest of the paper, I present general results that apply to every crisis bargaining game with uncertain stakes. Before proceeding to study the unrestricted class where states may bargain over both shares and transfers, this section studies crisis bargaining games in which settlements are restricted to only allocate shares or transfers, respectively. Although share and transfer protocols are equivalent in games with commonly known stakes, the following analysis reveals a significant difference between these crisis resolution technologies when the stakes are uncertain.

5.1 Share protocols

I begin by examining a subclass of crisis bargaining games in which settlements may only divide or allocate shares of the object, precluding an exchange of transfers between states in the form of side payments, compromises on other issues, or other external compensation mechanisms. I refer to this subclass as *share-protocol* crisis bargaining games.

Definition 3 (Share Protocol)**.** *A crisis bargaining game* Γ *uses a share protocol if and only if, for every* $a \in A$ *and all* $(\pi, x, t) \in \text{supp}(\gamma(a))$*,* $t = 0$ *whenever* $\pi = 0$*.*

The main result for share-protocol crisis bargaining games gives us reason to be optimistic about the prospects of peace when stakes are uncertain: always-peaceful equilibria can exist. In fact, always-peaceful equilibria in share-protocol games are generically easier to achieve with uncertain stakes than with uncertainty about other primitives.

To see why, consider an example of a simple share-protocol crisis bargaining shown in Figure 2. In the game, states observe their private types and simultaneously choose whether to fight or settle. They receive their war payoffs $w_i(\theta)$ if at least one chooses to fight, and if they both choose to settle, state 0 receives an x share of $v(\theta)$ whereas state 1 receives the complementary $1-x$ share, for a fixed $x \in [0, 1]$. Then, always-peace requires an x large enough to guarantee every type of state 0 their expected war payoff, but small enough to guarantee every type of state 1 their expected war payoff. Because high types have the largest expected war payoffs, this can be satisfied by a settlement x such that $c_1/V_1(\bar{\theta}_1) > x - p > -c_0/V_0(\bar{\theta}_0)$. A peaceful settlement is generally possible; indeed, peace is guaranteed if shares are always distributed according to the known balance of power, $x = p$. This

$\theta_0 \sim F_0$

$\theta_1 \sim F_1$

(θ_0, θ_1)	fight	settle
fight	$w_0(\theta), w_1(\theta)$	$w_0(\theta), w_1(\theta)$
settle	$w_0(\theta), w_1(\theta)$	$v(\theta)x, v(\theta)(1-x)$

Figure 2: Example of a share-protocol crisis bargaining game with uncertain stakes. State 0 (row) and 1 (column) simultaneously choose to fight or settle. Dashed boxes reflect state uncertainty about their opponent's type. Fighting yields war payoffs, whereas settling returns a share of the object corresponding to a fixed division x.

simple example admits an always-peaceful equilibrium, so there exist share-protocol crisis bargaining games that admit always-peaceful equilibria.

Proposition 1. *Suppose power p and resolve (c_0, c_1) are common knowledge, but there are uncertain stakes v. Then, there exists an incentive-compatible direct mechanism δ such that, for all $\theta \in \Theta$, $\pi = 0$ and $t = 0$ for all $(\pi, x, t) \in \operatorname{supp}(\delta(\theta))$. Therefore, there exists a share-protocol crisis bargaining game that admits an always-peaceful equilibrium.*

Always-peace can be achieved despite uncertain stakes. A natural way for this to occur is if a protocol can be instituted to allocate shares of the object in proportion to the known distribution of power between states. In this way, there is a clear similarity between the case of uncertain stakes under a share-protocol and the case of uncertainty about resolve (e.g., see Proposition 3 of Fey and Ramsay, 2011). Because power is commonly known, this division is certain to leave both states better off than war, regardless of what the true stakes are.

Even more optimistically, a wider range of always-peaceful divisions can usually be facilitated with uncertain stakes than with uncertain resolve. This is clear even in our example game, as we do not require shares exactly reflect the distribution of power for always-peace. Instead, a settlement that slightly favors one side or the other is permissible with always-peace as long as it remains within an acceptable range to satisfy the most demanding type of each state. For games in the share-protocol subclass in general, the interim participation constraint from Lemma 2 requires that each state expect to receive at least as much value from their share as they would from war, $\mathrm{E}_{\theta_j}[v(\theta)x_i^\delta(\theta) \,|\, \theta_i] \geq V_i(\theta_i)p_i - c_i$ for all $\theta_i \in \Theta_i$.

However, this condition is not sufficient and while there is some flexibility in the determination of the final division, the mapping from type pair to settlements is highly constrained. In fact, *none* of the variation in a state's interim expected utility across types can be attributable to changes in the equilibrium share allocation.

The variation is carried by two channels: a direct channel through the stakes of the dispute, and an indirect channel through beliefs about an opponent's private information. On the direct channel, each side can expect a higher utility when their type increases, simply because they expect the dispute to involve a more valuable object. Less straightforwardly, a marginal increase in a state's type can also change their beliefs about their opponent's type, which also affects their expectation of the object's value. Whether this serves to push a state's interim expected utility up or down depends on the sign of the correlation between private information. Accordingly, the indirect channel vanishes under independent types. Corollary 1 (Appendix A.3) presents the formal statement.

The constraint has substantive implications for the kinds of share protocols that can generate always-peace: any systematic linkage between a state's reported type and the size of their share allocation will violate incentive compatibility. In particular, always-peace is impossible under any share protocol that systematically rewards some types (e.g., those that expect higher stakes) with larger allocations than others. What survives is a narrow set of share allocation rules that do not create the incentive to misrepresent one's private information, such as a constant settlement at the power-balanced share.

Thus, while Proposition 1 demonstrates that always-peace is possible in a share-protocol crisis bargaining game, the only way to achieve always-peace in practice is to remove most of the resolution mechanism's flexibility to respond to reports. Mechanisms that choose different shares across reports in an effort to meet each type's unique demands are exactly the ones that fail.

5.2 Transfer protocols

In this section, I explore a subclass of bargaining games where states must agree to an exchange in which one state retrieves the entire object in lieu of side payment transfers. When the value of the object is known, transfer protocols are indistinguishable from share protocols discussed in the previous section: receiving x share of an object of value v is equivalent to receiving the object and extending a transfer equal to $v(1-x)$. However, as this section demonstrates, the distinction has important consequences when states are uncertain about the value of the object.

Definition 4 (Transfer Protocol)**.** *A crisis bargaining game* Γ *uses a transfer protocol if and only if, for every* $a \in A$ *and* $(\pi, x, t) \in \text{supp}(\gamma(a))$*,* $x_i = 1$ *for a given state* i *whenever* $\pi = 0$*.*

At this time, we make an additional assumption on the distribution of private types. In particular, we want to ensure there is sufficient uncertainty about the stakes of the dispute. If private information is perfectly correlated, for example, then each state i of type θ_i can precisely infer their opponent j's type θ_j. In turn, the true value of the object $v(\theta_i, \theta_j)$ is revealed by each side's private information, rendering the environment at hand one of essentially complete information. This screening effect of correlation defeats the point of our exercise in studying crisis bargaining when the stakes are uncertain. Accordingly, in my analysis of transfer protocols and one-sided indivisibilities, I proceed with an assumption that private types are drawn independently. This assumption is common in previous work, most notably in Fey and Ramsay (2011). In a later section, I show that correlated information can make always-peace even more difficult to achieve, as long as the screening effect just described is sufficiently weak.

Assumption 2 (Independent Types)**.** $F(\theta) = F_0(\theta_0)F_1(\theta_1)$.

$\theta_0 \sim F_0$

$\theta_1 \sim F_1$

(θ_0, θ_1)	fight	settle
fight	$w_0(\theta), w_1(\theta)$	$w_0(\theta), w_1(\theta)$
settle	$w_0(\theta), w_1(\theta)$	$v(\theta) - t, t$

Figure 3: Example of a transfer-protocol crisis bargaining game with uncertain stakes. State 0 (row) and 1 (column) simultaneously choose to fight or settle. Dashed boxes reflect state uncertainty about their opponent's type. Fighting yields war payoffs, whereas settling returns state 0 with the object in exchange for a fixed transfer t.

Let us now consider a simple example of a transfer-protocol crisis bargaining game, shown in Figure 3. As in the prior example, states observe their private type and simultaneously choose whether to fight or settle. If at least one chooses to fight, each state receives their war payoff. Unlike the previous example, if both states choose to settle, state 0 receives the object and extends a fixed transfer t to state 1.

First, given their opponent 0 is settling, state 1 will prefer to settle if and only if the transfer they receive is large enough to compensate them for their war payoff, $t \geq \mathrm{E}_{\theta_0} w_1(\theta)$. On the other hand, for state 0 to choose to settle when their opponent 1 is settling, the transfer they extend must be smaller then the additional value they receive from peacefully acquiring the object, $t \leq \mathrm{E}_{\theta_1}[v(\theta) - w_0(\theta)]$. Always-peace requires that this holds for all types of state 0 and 1; most importantly, the fixed transfer must be consistent with the demand of the highest type of the receiving state and the willingness of the lowest type of the extending state. Therefore, in our example, such a transfer does not exist when the expected war payoff of state 1's high type is larger than their opponent's expected gain from settling peacefully. Then, we can see that if $c_0 + c_1 < p_1(\bar{V}_1 - \underline{V}_0)$, always-peace is impossible.

This condition is not unique to the simple example with simultaneous decisions and a fixed transfer. In fact, this same inequality is a sufficient condition for the impossibility of always-peace in any crisis bargaining game that involves trading the object to state 0 for a transfer. More generally, we can state the following result.

Proposition 2. *Suppose power p and resolve (c_0, c_1) are common knowledge, but there are uncertain stakes v. If Assumption 2 holds and*

$$c_0 + c_1 < p_j(\bar{V}_j - \underline{V}_i)$$

for a state i and opponent j, then there does not exist an incentive-compatible direct mechanism δ such that, for all $\theta \in \Theta$, $\pi = 0$ and $x_i = 1$ for all $(\pi, x, t) \in \mathrm{supp}(\delta(\theta))$. Therefore, there does not exist a transfer-protocol crisis bargaining game that admits an always-peaceful equilibrium.

Unlike those with share protocols, crisis bargaining games with transfer protocols fail to admit always-peaceful equilibrium. The difference comes from the way states can take advantage of their private information, and the asymmetry between the two states' bargaining positions in a transfer-protocol game becomes consequential.

With transfer protocols, the extending state i, in walking away with the object, captures the upside of more favorable private information directly. In contrast, state j, in walking away with a transfer, cannot. A transfer scheme that tries to compensate higher types of j with higher transfers would invite mimicry from low types, who would strategically exaggerate their demands to capture larger transfers, as well. Anticipating this, state i would be less willing to extend large transfers in the first place. Share protocols avoid this difficulty because, by dividing the object directly, states with high expectations for the stakes can expect to receive a larger payoff than states with lower expectations without receiving a larger share. The receiving state's transfer must therefore be constant in their private information. Corollary 2 (Appendix A.4) shows that the only way for the receiving state's interim utility to vary in their type is then through the probability of war, but this channel would also be shut down in any always-peaceful equilibrium.

The condition in Proposition 2 is a direct consequence of this fact. Once expected transfers are forced to be constant across the receiving state's types, the question of whether always-peace is possible reduces to whether a single transfer can simultaneously satisfy the most demanding type of the receiving state and the least willing type of the extending state. Given large enough variation in the expected stakes across different types, no constant transfer can satisfy all types of both states, and always-peace is impossible. This is not a statement that no peaceful settlement exists in principle. On the contrary, one must exist: when the value of the object is commonly known, a transfer of $t = vp_j$ is guaranteed to satisfy all parties in the same way a share allocation of $x = p$ does. The problem is that states do not know the stakes, and the asymmetric bargaining positions preclude any transfer scheme that can overcome each side's incentive to misrepresent their private information, leading to war with positive probability in every equilibrium of every possible crisis bargaining game.

The condition also depends on the costs of war being not too large relative to the variation in expected stakes across types. Of course, some upper bound is expected, as states never consider war as a viable alternative when the costs of war grow infinitely large. This is comparable to Fey and Ramsay's (2011, Proposition 5) condition for the impossibility of an always-peaceful equilibrium under uncertain power, and qualifies the empirical applicability of the result. Wars in which nuclear escalation is likely, for example, may involve expected costs so extreme that fighting is categorically off the table; though, war never occurs on the path of play in these cases, so there is no puzzle of costly war to explain.

6 Bargaining over shares and transfers

We have so far established that always-peace is possible in crisis bargaining games of the share-protocol subclass but not in those of the transfer-protocol subclass. I now consider every possible crisis bargaining game. This general case covers any possible protocol, where the terms of peace that may include either an allocation of shares, an exchange of transfers, or both. Proposition 1 has already established that a game of this general class can admit an always-peaceful equilibrium. This section thus focuses on whether transfers can be useful to facilitate peace agreement when share allocations that are satisfactory to guarantee a mutually acceptable settlement are not available.

As seen with share protocols, always-peace can be guaranteed without transfers through an allocation of shares that sufficiently reflect the known balance of power. Then, what if such divisions are not possible? Can transfers help? I explore these questions by focusing on environments in which some share allocations are not possible. These settings have what is essentially a de facto indivisibility. I remain agnostic about the source of any indivisibility; it is not relevant if it reflects a literal inability to divide the object, an institutional inability to coordinate and reach particular divisions, or something else. All that matters is that, one way or another, sides cannot reach peaceful settlements that entail those share allocations.

The analysis proceeds by separately examining one- and two-sided indivisibilities. A crisis bargaining game is said to have a one-sided indivisibility when the set of infeasible shares prevents one party from recovering the entire object. A two-sided indivisibility is one where the set of infeasible shares constitutes an interior portion of the bargaining space, allowing for settlements that could, in principle, allocate the entire object to either side. As we will see, under mild conditions, it is impossible to construct a crisis bargaining game with a one-sided indivisibility that admits an always-peaceful equilibrium. In contrast, two-sided indivisibilities permit more flexibility; however, even these settings fail to allow for always-peace under a more rigorous robustness condition that peace not only needs to be reached in the first place, but maintained ex post.

6.1 One-sided indivisibilities

Let us first focus on any crisis bargaining game with a one-sided indivisibility. These are cases in which it is not feasible to implement settlements with shares that allocate extreme amounts of the object to one side or another. For example, this may arise in situations where one state refuses to peacefully forfeit too much of the object due to its importance for their sovereignty and continued survival. Figure 4 presents an example.

Definition 5 (One-sided Indivisibility)**.** *A crisis bargaining game* Γ *has a one-sided indivisibility if and only if there exists an* $\bar{x} \in [0, 1]$ *such that, for a given state* i *and every* $a \in A$*,* $x_i \geq \bar{x}$ *for all* $(\pi, x, t) \in \text{supp}(\gamma(a))$*.*

There are two details to note. First, to reduce the notational burden, I adopt the convention that state i is the side unable to peacefully concede extreme amounts of the object and their opponent j cannot peacefully recover the full object, so that $\bar{x}$ is the minimum share that can be peacefully allocated to i and hence $1 - \bar{x}$ is the maximum share that can be peacefully allocated to j, without loss of generality. Second, by this definition, it is clear that every crisis bargaining game with a one-sided indivisibility such that $\bar{x} = 1$ is a game in the transfer-protocol subclass.

The tension created by a one-sided indivisibility is straightforward: to achieve always-peace whenever such an indivisibility exists with corresponding threshold $\bar{x}$ for some state i, then the opponent j must always be willing to accept a settlement in which they are allocated a share of no

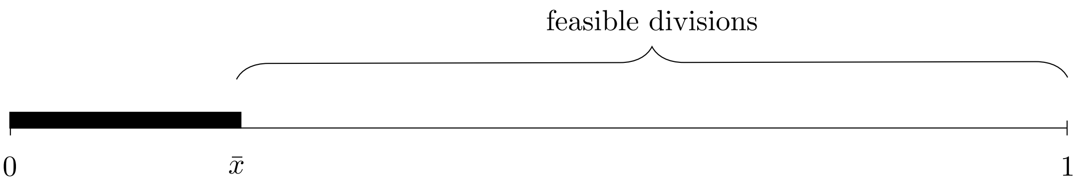


Figure 4: Example of a one-sided indivisibility. *Note:* There is a one-sided indivisibility when, for all $a \in A$, every realized outcome with $\pi = 0$ in the support of $\gamma(a)$ leaves a state i with at least $\bar{x}$ of the object. For this illustration, $i = 0$.

more than $1 - \bar{x}$. But some types of the opponent may value the object very highly and be unwilling to accept shares below $1 - \bar{x}$ without adequate compensation in the form of transfers. Can transfers help facilitate peace?

For a transfer scheme to be implementable within an equivalent direct mechanism, it needs to satisfy both the most pessimistic type of state i that expects the lowest stakes and the type of the most optimistic type of opponent j that expects the highest stakes. Transfers can vary type pair to type pair; however, they must maintain incentive compatibility—that is, as the transfer received by one type of opponent j becomes much larger than that received by another type of j, the types receiving less will mimic the behavior of the type receiving more unless these incentives are balanced by their corresponding expected shares of the object. If these incentives cannot be balanced using shares and transfers, there will be a risk of war in equilibrium. The following result characterizes sufficient interim conditions that rule out the possibility of an always-peaceful equilibrium in every crisis bargaining game.

Theorem 1. *Suppose power p and resolve (c_0, c_1) are common knowledge, but there are uncertain stakes v. If Assumption 2 holds and there is a one-sided indivisibility with $\bar{x} \in [0, 1]$ such that*

$$c_0 + c_1 < (\bar{x} - p_i)(\bar{V}_j - \underline{V}_i) - (1 - \bar{x})(\mathrm{E}v - \underline{V}_j) \tag{2}$$

for a state i and opponent j, then there does not exist an incentive-compatible direct mechanism δ such that, for all $\theta \in \Theta$, $\pi = 0$ and $x_i \geq \bar{x}$ for all $(\pi, x, t) \in \mathrm{supp}(\delta(\theta))$. Therefore, there does not exist a crisis bargaining game with such a one-sided indivisibility that admits an always-peaceful equilibrium.

Inequality (2) provides the main condition that prevents always-peace with a one-sided indivisibility. It is unsurprising that we cannot rule out the existence of a crisis bargaining game with an always-peaceful equilibrium if the costs of war are too large (of course, war would never occur if the costs of war are increased to infinity). However, the right-hand side of the inequality demonstrates that this condition is fairly mild.

In particular, the first component of the right-hand side corresponds to the compensation problem states face in attempting to resolve the dispute by dividing the pie. The quantity $\bar{x} - p_i$ reflects the minimum distance a settlement's share must be from the power-balanced share. This interacts directly with $\bar{V}_j - \underline{V}_i$, the difference between the highest possible expected stakes of one state and the lowest possible expected stakes of another. Together, the first term captures the value extreme types of each side ascribe to the minimum imbalance in shares caused by the indivisibility. Intuitively, this is the size of the problem created by the inability to settle with certain shares.

Transfers can, in principle, be used to make the opponent j whole and overcome this imbalance in shares. However, as shown in the transfer-protocol subclass, transfers can create an incentive to mimic types that receive larger transfers if those same types do not also receive a sufficiently smaller share of the object, given beliefs about the value of those shares, to offset this force. The term $1 - \bar{x}$ gives the maximum share that can be allocated to the side that is shortchanged, receiving less than its power-balanced share. As such, it governs the flexibility any mechanism designer has to vary j's shares to maintain incentive compatibility. This interacts directly with the term $\mathrm{E}v - \underline{V}_j$, which governs how much additional value j attributes to the shares on average. If j typically values a share of the object much more than its lowest type, this gives the designer more flexibility in pairing high transfers with less favorable shares and vice versa.

This impossibility is driven by the fact that, regardless of what conflict resolution procedure is in place, private information about the stakes of the dispute induce states to trade off a risk of war for an opportunity to receive more lucrative settlements in peace. Therefore, it is not that always-peace is impossible in principle, but that there are no incentive-compatible mechanisms that can be instituted to generate peace with certainty in equilibrium.

An implication of Theorem 1 is that small indivisibilities can be an insurmountable obstacle to always-peace when there is a high power differential between the states. In particular, when the weaker side is arbitrarily weak, always-peace can be impossible even as the indivisibility becomes arbitrarily small. Whether or not such a situation is realized depends on the relationship between private information and the stakes of the dispute: it is most difficult to achieve always-peace when state i's lowest type expects very small stakes, the opponent j's highest type expects very large stakes, and j's lowest type expects stakes to be close to the ex-ante expectation.

One intuitive way for this to occur is if the stronger state's private information can unilaterally swing the stakes upward far beyond the ex-ante expectation. To see this, consider a simple example in which $v(\theta_i, \theta_j) = \theta_i + \theta_j^2$, where θ_i is distributed uniformly on $[1, 3]$ and θ_j is drawn to be 0 with probability 999/1000 and 20 with probability 1/1000. This example reflects the case in which, with very small probability, the object in dispute is a jackpot and opponent j alone has private knowledge of whether it is. Simple calculations reveal that the lowest expectation state i can have is 7/5, which is significantly less than the highest expectation state j can have, which is 402. Further, the ex-ante expectation is 12/5, which is not far from 2, the expected stakes of j's lowest type. Even with an indivisibility only marginally precluding peace at the power-balanced share, the right-hand side of inequality (2) can be large.

To pin down a specific case, take $p_i = 1/200$ and $\bar{x} = 1/100$. Then, the right-hand side of (2) becomes approximately 8/5. Even though the indivisibility is very small in absolute terms, only preventing settlements that allocate more than 99% of the object to opponent j, an always-peaceful equilibrium is not possible unless the total costs of war are so large as to exceed half of the ex-ante expected value of the object. Increasing the value of the unlikely jackpot further worsens the prospect

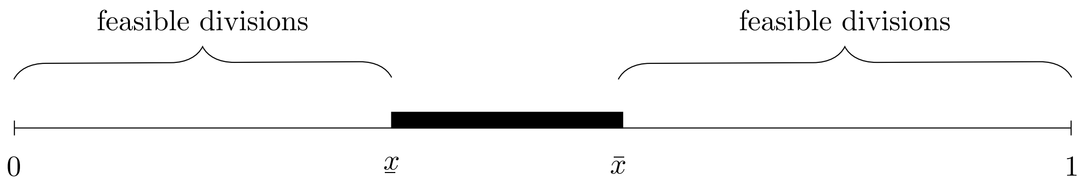


Figure 5: Example of a two-sided indivisibility. *Note:* There is a two-sided indivisibility when, for all $a \in A$, no realized outcome with $\pi = 0$ in the support of $\gamma(a)$ involves a share between $\underline{x}$ and $\bar{x}$.

of always-peace, requiring even greater total costs of war.

6.2 Two-sided indivisibilities

Next, let us consider indivisibilities that are "two-sided" in that extreme settlements are possible in each direction. In this case, it is possible for states to agree to peaceful settlements that allocate the entire object to either party. Figure 5 provides an illustration.

Definition 6 (Two-sided Indivisibility). *A crisis bargaining game* Γ *has a two-sided indivisibility if and only if there exists* $\underline{x}, \bar{x}$ *such that* $1 > \bar{x} > \underline{x} > 0$ *and, for every* $a \in A$ *and all* $(\pi, x, t) \in \text{supp}(\gamma(a))$, $x \notin (\underline{x}, \bar{x})$.

Unlike its one-sided counterpart, two-sided indivisibilities that preclude peaceful settlements at the power-balanced share do not fundamentally bias ownership of the object towards one side or the other. As a result, conflict resolution mechanisms have more flexibility in the face of a two-sided indivisibility. In particular, because the mechanism designer can allocate a more-than-power-balanced share to either side, it has an additional degree of freedom in varying shares across type profiles so that each state of each type expects a favorable share at least some of the time. Because peace is efficient, the mechanism designer can strategically design a settlement scheme so that the gains and losses across these different realizations of type profiles wash out on average, leaving each type of each state expecting more from peace than their war payoff.

To see why always-peace is generally possible in this case, consider a simple example. Suppose each side wins a war half of the time, but that shares that allocate more than 40% but less than 60% of the object to either side are infeasible; that is, $p = 0.5$ and $(\underline{x}, \bar{x}) = (0.4, 0.6)$. Then, even though the mechanism designer cannot evenly divide the object for any particular type profile realization, they can still implement a settlement scheme so that each type of each state expects an equivalent payoff from peace over all possible realizations. One such way the mechanism designer could do this is by simply flipping a coin and allocating 60% to the winner of the coin flip.

The following proposition formally states the general result.

Proposition 3. *Suppose power* p *and resolve* (c_0, c_1) *are commonly known, but there is uncertainty about stakes* v. *If there is a two-sided indivisibility given by any* $\underline{x} \in [0, 1)$ *and* $\bar{x} \in (\underline{x}, 1]$, *then there exists an incentive-compatible direct mechanism* δ *such that* $\pi = 0$ *for all* $(\pi, x, t) \in \text{supp}(\delta(\theta))$, *for all* $\theta \in \Theta$. *Therefore, there exists such a crisis bargaining game with such a two-sided indivisibility that admits an always-peaceful equilibrium.*

To overcome a two-sided indivisibility, the proof uses an example in which the mechanism designer implements a randomization device over the final settlement. For example, given a reported type pair $\tilde{\theta}$, suppose that the mechanism designer chooses the settlement randomly between settlements in the following way. With probability q, they select a share of $\bar{x}$ and transfer $-v(\tilde{\theta})(\bar{x}-p)$, or else they select a share $\underline{x}$ and a transfer $v(\tilde{\theta})(p-\underline{x})$. If the probability of selecting the first settlement is chosen so that $q\bar{x}+(1-q)\underline{x}=p$, then the expected peace payoff for every type is $V_i(\theta_i)p_i$ regardless of their report. In effect, the lottery over settlements recovers the interim expected utilities associated with the power-balanced share that the indivisibility prohibits. This "randomization-device" mechanism is a variation of the ones proposed by Fearon (1995), Powell (2006), and Wagner (2010).

This example is consistent with the crisis bargaining game form as we have defined it, but only because we have opted for an inclusive definition that does not preclude credible commitments. In particular, the mechanism here works because of its stochastic nature: states only need to expect peace to be better on average, even though some types would have to honor realized settlement terms they know to be worse than war. Crisis bargaining games rarely feature this kind of commitment power, as the anarchic environment leaves states without the external enforcement that would be required to bind them to unfavorable realizations. When settlement terms are revealed that leave one state worse off than war, they can often abandon the agreement and fight.

An interesting question to explore regarding two-sided indivisibilities, then, is not whether states can agree to a peace they expect to be better than war on average, but whether any such peace can be maintained once the terms of settlement are realized. To study this, I introduce the following notion of robustness, which precludes states from committing ex ante to honor lottery realizations of peace settlements.

Definition 7 (Robustness). *A direct mechanism δ is called robustly implementable if and only if, for each state i, it satisfies:*

1. *No randomization over the settlement: for all $\theta \in \Theta$, there exists a $(\pi^{\delta}(\theta), x^{\delta}(\theta), t^{\delta}(\theta)) \in \Omega$ such that* $\text{supp}(\delta(\theta)) = \{(\pi^{\delta}(\theta), x^{\delta}(\theta), t^{\delta}(\theta))\}$.

2. *Ex-post incentive compatibility: for all $\theta_i, \tilde{\theta}_i \in \Theta_i$, $\theta_j \in \Theta_j$,*

$$u_i^{\delta}(\theta_i; (\theta_i, \theta_j)) \geq u_i^{\delta}(\tilde{\theta}_i; (\theta_i, \theta_j)). \tag{EPIC}$$

3. *Ex-post voluntary agreements: for all $\theta_i \in \Theta_i$, $\theta_j \in \Theta_j$,*

$$u_i^{\delta}(\theta_i; (\theta_i, \theta_j)) \geq w_i(\theta_i, \theta_j). \tag{EPVA}$$

An equilibrium σ^* of a crisis bargaining game is called robust if and only if the outcome it induces admits a robustly implementable equivalent direct mechanism.

Although the conditions of robustness are more difficult to jointly satisfy than the baseline conditions, they are nonetheless fairly mild and consistent with the vast majority of crisis bargaining games. Let us go through each of them in turn.

The first condition rules out randomization over the settlement. This means that robustly implementable mechanisms must deliver a specific settlement term for any type pair, ruling out the possibility that states could commit to honoring an outcome of a lottery over peace deals ex ante as in the previous example of Proposition 3. This restriction is the default of previous game-free analyses, most notably in Fey and Ramsay (2011), which implicitly restricts their formulation of the crisis bargaining game form to those that admit deterministic mechanisms. Kenkel and Schram (2025) likewise assume deterministic settlements throughout.

Second, ex-post incentive compatibility (EPIC) strengthens the baseline incentive compatibility condition presented by Definition 1. Incentive compatibility requires that each state must weakly prefer to truthfully report their type given their beliefs about the stakes of the dispute. In contrast, EPIC requires that states weakly prefer to truthfully report their type *given* the stakes of the dispute. This is a common assumption in a growing literature on "robust" mechanism design that develops results free of any particular specification of beliefs. In particular, Bergemann and Morris (2016) show that this stronger condition is equivalent to the mechanism being incentive compatible under every possible information structure governing beliefs about each other's types.

Third, the ex-post voluntary agreements (EPVA) criterion strengthens the baseline requirement for voluntary agreements from Assumption 1. Under our VA criterion, consistency with the crisis bargaining game form requires only that there exists an action available to each state which, if chosen, guarantees an expected interim payoff at least as good as their expected interim war payoff. The EPVA criterion, on the other hand, requires that the equilibrium payoff to every type of each state is at least as good as that state's war payoff, given their opponent's type. This means that no state can find themselves "stuck" with a peaceful settlement that is worse for them than war.

Together, the three conditions help us understand not when states can reach peace with certainty, but when states can maintain peace with certainty. Then, always-peace must be a stable outcome not just on average, but at every realization the mechanism may produce. The following result establishes that, under these robustness conditions, two-sided indivisibilities preclude always-peaceful equilibria in every crisis bargaining game with uncertain stakes.

Theorem 2. *Suppose power p and resolve (c_0, c_1) are common knowledge, but there are uncertain stakes v. Further, let v be strictly increasing in both of its arguments and Θ_i be a compact interval for each state i. Then, if there is a two-sided indivisibility given by $\underline{x} \in [0,1)$ and $\bar{x} \in (\underline{x}, 1]$ such that*

$$c_0 + c_1 < \frac{(\bar{x} - p)(p - \underline{x})}{\bar{x} - \underline{x}} \max_i \left\{ \max_{\theta_j} [v(\bar{\theta}_i, \theta_j) - v(\underline{\theta}_i, \theta_j)] \right\}, \tag{3}$$

there does not exist a robustly implementable direct mechanism δ such that $\pi = 0$ for all $(\pi, x, t) \in \text{supp}(\delta(\theta))$, for all $\theta \in \Theta$. Therefore, there does not exist a crisis bargaining game with such a two-sided indivisibility that admits a robust always-peaceful equilibrium.

Under the robustness conditions, the mechanism designer no longer has the lottery mechanism available to them, and they must choose a peaceful settlement scheme so that shares never fall within the indivisibility, transfers compensate states that receive unfavorable shares, and states have the incentive to truthfully report their type. In this setting, EPIC forces share allocations in which higher types receive more favorable shares. The intuition is that transfers can compensate for differences in shares only when the share allocation is monotone in type. Due to the two-sided indivisibility, share allocations must "jump" from weakly below $\underline{x}$ to weakly above $\bar{x}$ as state 0's type increases past the threshold, and vice versa for state 1.

Consider types just above and below the threshold. Both must prefer their assigned settlement to the other's and, because the higher type values the object more, incentive compatibility requires the difference between what each receives in peace to grow as the stakes of the dispute rise. But the EPVA criterion puts a constraint on that difference. Because every type of every state must always prefer peace to war, the difference in peace payoffs between any two types cannot exceed the total costs of war. When there is sufficient variation in stakes across types, EPIC demands more variation in states' peace payoffs than EPVA allows.

Despite the strengthened robustness conditions, it is worth reiterating that the impossibility result of Theorem 2 holds under fairly mild conditions. In fact, the condition reveals that impossibility may result even with arbitrarily small indivisibilities around the power-balanced share, as long as there is sufficient variation in the object's value. Moreover, unlike the preceding impossibility results, Theorem 2 does not make any assumptions about the distribution of private information; indeed, the criteria for a robust always-peaceful equilibrium cannot be jointly satisfied under any joint distribution F.

7 Correlated information

This section examines the effect of correlated private information, and demonstrates that the main impossibility logic extends to settings in which one state's type provides them with additional information about their opponent's type. I identify two countervailing forces: one in *screening*, in which more information about the opponent's type helps facilitate peaceful agreement, and another through *amplification*, in which states with extreme signals intensify their demands, making it more difficult to successfully reach an agreement and avoid war.

With a few additional regularity conditions (e.g., supermodularity of v), one could derive general statements about the impossibility of always-peace under correlated information, comparable to Theorems 1 and 2. However, because we are primarily interested in demonstrating the less-intuitive amplification effect, and that it can outweigh the more-intuitive screening effect and exacerbate the problem, I choose to focus this section on a special case. To begin, let us specify the distribution of private information.

Assumption 3 (Correlated Types)**.** *Types θ_0, θ_1 are drawn from the joint density*

$$f(\theta_0, \theta_1; \lambda) = \frac{1}{4}(1 + \lambda\theta_0\theta_1)$$

on Θ, with $\Theta_0 = \Theta_1 = [-1, 1]$ and $\lambda \in [0, 1]$.

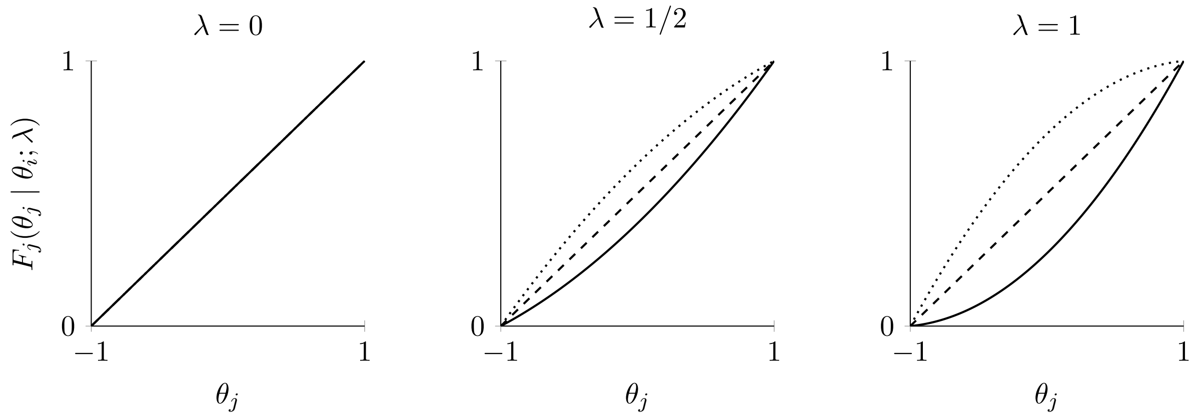


Figure 6: Examples of the conditional CDF $F_j(\theta_j \,|\, \theta_i; \lambda)$ implied by Assumption 3. *Note:* From left to right, the plots show distributions under zero, moderate, and high correlation, reflected by changes in the dependence parameter λ. The dotted, dashed, and solid lines reflect the distribution of θ_j given a low, moderate, and high value of θ_i, respectively.

By this assumption, the degree of correlation is measured by a dependence parameter λ, with $\lambda = 0$ corresponding to independent types and larger λ implying a higher correlation between each state's type; specifically, $\text{corr}(\theta_0, \theta_1) = \lambda/3$. Then, as λ increases, a state's own private information reveals more about their opponent's private information. While this specification focuses on positively correlated information, comparable results can be attained with negatively correlated information, as well. Figure 6 presents examples of the implied conditional cumulative distribution.

This particular specification is useful for our purposes for two reasons. First, a single parameter λ captures the degree of correlation, with independent types recovered at $\lambda = 0$. Second, although the joint and conditional densities change in λ, the marginal distribution of each state's own type does not: each θ_i is uniformly distributed on $[-1, 1]$ for every λ. Both aspects matter for us. The conditional's λ-dependence captures the screening channel, while the marginal's λ-invariance guarantees that only the linkage in private information changes in λ, not the distribution from which each state's own type is drawn. To put it concretely, if states privately know how much gold is on their side of a territorial dispute, increases in λ make gold on one side more predictive of gold on the other side without changing the amount of gold expected on either side ex ante.

Next, let us further specify how private information relates to the dispute's stakes.

Assumption 4. $v(\theta_0, \theta_1) = \phi(\theta_0 + \theta_1)$ *where* $\phi : [-2, 2] \to \mathbb{R}$ *is continuously differentiable, strictly increasing, and strictly convex, with* $\phi(-2) > \max_i c_i/p_i$.

Letting the value of the object depend on the sum $\theta_0 + \theta_1$ is a simplifying assumption that reduces the two-dimensional type profile to a single scalar. More importantly, the assumption of strict convexity implies complementarities between types, meaning that the marginal value of a higher type is increasing in the type of the other side. In the gold example, this essentially implies a scale economy: the more gold on one side, the greater the value of finding a little more gold the other side. We would expect this if, for example, the marginal cost of extraction decreases in the size of the deposit. The other conditions simply re-establish existing assumptions on v.

Given this, let us consider a transfer-protocol crisis bargaining game. To adapt notation for correlated private information with dependence parameter λ, let $V_i(\theta_i;\lambda) = \int v(\theta_i, z) f_j(z \,|\, \theta_i;\lambda)dz$ denote state i of type θ_i's interim expected stakes, and let $\mathrm{E}v(\lambda) = \int v(z)f(z;\lambda)dz$ denote the unconditional expected stakes. Then, we can derive an analogue to Proposition 2's impossibility: if $c_0 + c_1 < p_j(\mathrm{E}v(\lambda) - V_i(0;\lambda))$ for some state i and an opponent j, there does not exist a transfer-protocol crisis bargaining game with uncertain stakes that admits an always-peaceful equilibrium. The formal statement is presented as Proposition 5 (Appendix A.12).

Correlation gives rise to two countervailing effects, each captured in the quantity $\mathrm{E}v(\lambda) - V_i(0;\lambda)$. First, the screening effect enables each state to, upon seeing their own type, make stronger inferences about their opponent's type, and hence the value of the object. This channel operates through the conditional density $f_j(\theta_j \,|\, \theta_i;\lambda)$, which assigns higher probability to θ_j with the same sign as θ_i. Under Assumption 3, the screening effect is strong at extreme types, but weak for moderate types. Importantly, the conditional density $f_j(\theta_j \,|\, 0;\lambda)$ does not depend on λ, so the screening effect is not in play for the median type $\theta_i = 0$. This fact is essential to the structure of the argument, as the median type establishes the bound on the receiving state's expected transfer.

Second, the amplifying effect operates through $\mathrm{E}v(\lambda)$. In particular, correlation increases the joint probability that both states draw a similar type, and Assumption 4 guarantees that there are complementarities in v, meaning that the effect a rise in one state's type has on the stakes of the dispute is increasing in their opponent's type. By Lemma 8 (Appendix A.10), this means that $\mathrm{E}v(\lambda)$ is strictly increasing in λ. Therefore, because $V_i(0;\lambda)$ is fixed under Assumption 3 and $\mathrm{E}v(\lambda)$ is strictly increasing in λ under Assumption 4, the quantity of interest strictly increases in λ. This means that, under these conditions, the amplifying effect dominates the screening effect, and more correlation makes it harder for crisis bargaining games to admit always-peaceful equilibria. The result is stated formally below.

Proposition 4. *Suppose Assumptions 3 and 4 hold, and consider transfer-protocol crisis bargaining games with uncertain stakes. If* $V_j(1;0) - V_i(-1;0) < \mathrm{E}v(1) - V_i(0;1)$ *for a state* i *and opponent* j*, then there exist primitives* $(p, c_0, c_1) \in (0,1) \times \mathbb{R}^2_+$ *and a unique* $\bar{\lambda} \in (0,1)$ *such that no game admits always-peaceful equilibria under any* $\lambda > \bar{\lambda}$ *but there do exist games that admit always-peaceful equilibria under* $\lambda = 0$*.*

The result qualifies the conventional logic that less uncertainty about an opponent's type promotes peace in crisis bargaining. It is true that correlation can give each state more information, but it can also amplify the demands of high types by reinforcing their belief that the stakes of the dispute are high. Thus, additional information does not necessarily facilitate peace.

8 Conclusion

This paper examines crisis bargaining games when there is an unknown common value for the object, which I call the stakes of the dispute. I introduce a formulation of the crisis bargaining game form, extending previous work from Fey and Ramsay (2011), and use a game-free approach to study every possible game in this class. I show that, while always-peaceful equilibria can exist under share protocols (Proposition 1), transfer protocols fail to do so (Proposition 2). This runs against conventional wisdom that suggests transfers in the form of side payments and issue linkages can rescue peaceful settlements in the face of dispute indivisibilities.

I then show that transfer-protocol games are a special case of one-sided indivisibilities and extend the impossibility result to the general case (Theorem 1). Two-sided indivisibilities permit the mechanism designer more flexibility, and always-peace remains possible by randomizing over peaceful settlements (Proposition 3). However, under stronger yet still fairly mild robustness conditions, I show that always-peaceful equilibria do not survive in two-sided indivisibilities either (Theorem 2). Finally, I extend the analysis to consider correlated private information. I identify countervailing screening and amplification effects, derive an impossibility condition under special conditions, and show that correlation can make always-peace more difficult to achieve when the amplification effect outweighs the screening effect (Proposition 4).

The findings have implications for what types of disputes are hardest to resolve peacefully. If negotiating a settlement requires an exchange of transfers, such as a concession on another issue, then the resolution procedures are especially susceptible to failure. This is because states cannot denominate transfers in the unknown value of the object, and so any state compensated with transfers either misses out on upside when the stakes are high, or invites lower types to mimic their report. Extending that logic, even small indivisibilities can pose a significant barrier to peace, especially when there is a highly asymmetric power balance. Finally, the results have implications for institutional design: institutions that seek to improve information can counterintuitively undermine the prospects for peace, as better information about the stakes of the dispute can make states with extreme beliefs less willing to compromise.

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Appendix for "Crisis Bargaining Over What?"

Contents

A Auxiliary Results

The paper's main results depend on the following auxiliary results. Lemma 1 applies broadly to every crisis bargaining game with uncertainty about any collection of primitives (v_0, v_1, p, c_0, c_1). In all other statements, I assume a crisis bargaining game with uncertain stakes such that power p and resolve (c_0, c_1) are common knowledge, but the value of the object is common $v_0 = v_1 = v$ and unknown to both sides.

All formal statements are numbered by the order in which they are presented. To make the logical structure between the main and auxiliary results explicit, Proposition 1 can be proved after Lemma 2, Theorem 1 and Propositions 2 and 3 after Lemma 4, Theorem 2 after Lemma 6, and Proposition 4 after Proposition 5.

A.1 Lemma 1

Lemma 1. *Let δ be an incentive-compatible direct mechanism such that $U_i^\delta(\theta_i;\theta_i) \geq \mathrm{E}_{\theta_j}[w_i(\theta)\,|\,\theta_i]$ for each state i and all $\theta_i \in \Theta_i$. Then, there exists a crisis bargaining game Γ and a Bayesian Nash equilibrium σ^* of Γ of which δ is an equivalent direct mechanism.*

Proof. Take any incentive-compatible direct mechanism δ such that $U_i^\delta(\theta_i;\theta_i) \geq \mathrm{E}_{\theta_j}[w_i(\theta)\,|\,\theta_i]$ for each state i and all $\theta_i \in \Theta_i$. Construct the following game.

Let each state i have actions $A_i = \Theta_i \cup \{W_i\}$ where W_i is state i's action to fight a war, and define the outcome function by $\gamma(a) = \delta(a)$ if $a \in \Theta$ and $\gamma(a)(\{(1,x,t)\}) = 1$ for any $(x,t) \in [0,1] \times \mathbb{R}$ otherwise. Let the war action by either state yield war payoffs against any action of their opponent, therefore satisfying the VA criterion of Assumption 1.

Now consider $\sigma^*(\theta) = \theta$. Fix a state i, type θ_i, and a deviation $\tilde{a}_i \in A_i$. If $\tilde{a}_i = \tilde{\theta}_i \in \Theta_i$, then the deviation yields $U_i^\delta(\tilde{\theta}_i;\theta_i) \leq U_i^\delta(\theta_i;\theta_i)$ by incentive compatibility of δ. If $\tilde{a}_i = W_i$, then the deviation yields $\mathrm{E}_{\theta_j}[w_i(\theta)\,|\,\theta_i] \leq U_i^\delta(\theta_i;\theta_i)$ by the antecedent. Thus, σ^* is a Bayesian Nash equilibrium and $\gamma \circ \sigma^*$ induces the same distribution over outcomes as δ. □

A.2 Lemma 2

Lemma 2 (Interim Participation Constraint)**.** *If a direct mechanism δ satisfies $\pi = 0$ for all $(\pi, x, t) \in \mathrm{supp}(\delta(\theta))$, $\theta \in \Theta$, and inequality (1) holds for each state i and all $\theta \in \Theta$, then $U_i^\delta(\theta_i;\theta_i) \geq V_i(\theta_i)p_i - c_i$ for each state i and all $\theta_i \in \Theta_i$.*

Proof. Fix a state i of type θ_i. Taking expectations over the realized posterior $\mu_i(\theta_i)$ given θ_i,

$$\mathrm{E}_{\mu_i(\theta_i)}\left[\mathrm{E}_{\theta_j}\left[v(\theta)x_i^\delta(\theta) + t_i^\delta(\theta) - w_i(\theta)\,|\,\mu_i(\theta_i)\right]\right] = \mathrm{E}_{\theta_j}\left[v(\theta)x_i^\delta(\theta) + t_i^\delta(\theta) - w_i(\theta)\,|\,\theta_i\right]$$

by the law of iterated expectations. Inequality (1) therefore implies

$$\mathrm{E}_{\theta_j}[v(\theta)x_i^\delta(\theta) + t_i^\delta(\theta)\,|\,\theta_i] \geq \mathrm{E}_{\theta_j}[w_i(\theta)\,|\,\theta_i]. \tag{A.1}$$

By definition, the right-hand side of inequality (A.1) is equal to $V_i(\theta_i)p_i - c_i$ and, because $\pi = 0$ for all $(\pi, x, t) \in \text{supp}(\delta(\theta))$ and all $\theta \in \Theta$, the left-hand side of (A.1) is $U_i^\delta(\theta_i; \theta_i)$. □

A.3 Corollary 1

Corollary 1. *Suppose F admits a conditional density $f_j(\cdot \,|\, \theta_i)$ that is continuously differentiable in (θ_i, θ_j). In any always-peaceful equilibrium σ^* of a share-protocol crisis bargaining game,*

$$\frac{dU_i^{\sigma^*}(\theta_i)}{d\theta_i} = \int_{\Theta_j} \frac{\partial v}{\partial \theta_i}(\theta_i, z)x_i^\delta(\theta_i, z)f_j(z \,|\, \theta_i)dz + \int_{\Theta_j} v(\theta_i, z)x_i^\delta(\theta_i, z)\frac{\partial f_j(z \,|\, \theta_i)}{\partial \theta_i}dz$$

for each state i and all $\theta_i \in \Theta_i$.

Proof. In any always-peaceful equilibrium of a share-protocol game, $\pi^\delta(\theta) = 0$ and $t^\delta(\theta) = 0$ for all $\theta \in \Theta$, so $U_i^\delta(\tilde{\theta}_i; \theta_i) = \int_{\Theta_j} v(\theta_i, z)x_i^\delta(\tilde{\theta}_i, z)dF_j(z \,|\, \theta_i)$. Because f_j is C^1 in θ_i, the partial derivative is

$$\frac{\partial U_i^\delta(\tilde{\theta}_i, \theta_i)}{\partial \theta_i} = \int_{\Theta_j} \frac{\partial v}{\partial \theta_i}(\theta_i, z)x_i^\delta(\tilde{\theta}_i, z)f_j(z \,|\, \theta_i)dz + \int_{\Theta_j} v(\theta_i, z)x_i^\delta(\tilde{\theta}_i, z)\frac{\partial f_j(z \,|\, \theta_i)}{\partial \theta_i}dz. \tag{A.2}$$

By incentive compatibility, $U_i^{\sigma^*}(\theta_i) = \sup_{\tilde{\theta}_i} U_i^\delta(\tilde{\theta}_i; \theta_i)$ and hence, by the envelope theorem (Milgrom and Segal, 2002, Theorem 2),

$$\frac{dU_i^{\sigma^*}(\theta_i)}{d\theta_i} = \left.\frac{\partial U_i^\delta(\tilde{\theta}_i; \theta_i)}{\partial \tilde{\theta}_i}\right|_{\tilde{\theta}_i = \theta_i}. \tag{A.3}$$

Evaluating equation (A.2) at $\tilde{\theta}_i = \theta_i$ and substituting into (A.3) yields the desired expression. □

A.4 Corollary 2

Corollary 2. *Suppose Assumption 2 holds and fix state i so that $x_i = 1$ for all $(0, x, t) \in \text{supp}(\delta(\theta))$, $\theta \in \Theta$. Then, in any equilibrium σ^* of a transfer-protocol crisis bargaining game,*

$$\frac{dU_i^{\sigma^*}(\theta_i)}{d\theta_i} = \int_{\Theta_j} \frac{\partial v}{\partial \theta_i}(\theta_i, z)(1 - p_j\pi^\delta(\theta_i, z))dF_j(z)$$
$$\frac{dU_j^{\sigma^*}(\theta_j)}{d\theta_j} = p_j \int_{\Theta_i} \frac{\partial v}{\partial \theta_j}(\theta_j, z)\pi^\delta(\theta_j, z)dF_i(z).$$

Proof. Fix a state i such that $x_i^\delta = 1$ in any peaceful settlement and note that, by Assumption 2, $dF(z \,|\, \theta_i) = dF_j(z)$ and $dF(z \,|\, \theta_j) = dF_i(z)$.

First, taking the partial derivative of $U_i^\delta(\tilde{\theta}_i; \theta_i)$ with respect to θ_i, we have

$$\frac{\partial U_i^\delta(\tilde{\theta}_i; \theta_i)}{\partial \theta_i} = \int_{\Theta_j} \frac{\partial v}{\partial \theta_i}(\theta_i, z)(1 - p_j\pi^\delta(\tilde{\theta}_i, z))dF_j(z). \tag{A.4}$$

By incentive compatibility $U_i^{\sigma^*}(\theta_i) = \sup_{\tilde{\theta}_i} U_i^\delta(\tilde{\theta}_i; \theta_i)$, and the envelope theorem (Milgrom and Segal, 2002, Theorem 2) gives $dU_i^{\sigma^*}/d\theta_i = \partial U_i^\delta/\partial\theta_i|_{\tilde{\theta}_i=\theta_i}$. Then, evaluating equation (A.4) at $\tilde{\theta}_i = \theta_i$ recovers the first expression.

Second, taking the partial derivative of $U_j^\delta(\tilde{\theta}_j;\theta_j)$ with respect to θ_j,

$$\frac{\partial U_j^\delta(\tilde{\theta}_j;\theta_j)}{\partial\theta_j} = p_j \int_{\Theta_i} \frac{\partial v}{\partial\theta_j}(\theta_j, z)\pi^\delta(\tilde{\theta}_j, z))dF_i(z). \tag{A.5}$$

By the same envelope theorem argument, evaluating (A.5) at $\tilde{\theta}_j = \theta_j$ yields the second expression. □

A.5 Lemma 3

Lemma 3. *If Assumption 2 holds, then for any equilibrium σ^* of any crisis bargaining game Γ with incentive-compatible equivalent direct mechanism δ, the interim expected utility for state with type θ_i must satisfy*

$$\frac{dU_i^{\sigma^*}(\theta_i)}{d\theta_i} = \int_{\Theta_j} \frac{\partial v}{\partial\theta_i}(\theta_i, z)[\pi^\delta(\theta_i, z)p_i + (1-\pi^\delta(\theta_i, z))x_i^\delta(\theta_i, z)]dF_j(z).$$

If σ^* is an always-peaceful equilibrium, $\pi^\delta(\theta) = 0$ for all $\theta \in \Theta$ and therefore

$$\frac{dU_i^{\sigma^*}(\theta_i)}{d\theta_i} = \int_{\Theta_j} \frac{\partial v}{\partial\theta_i}(\theta_i, z)x_i^\delta(\theta_i, z)dF_j(z).$$

Proof. By incentive compatibility, $U_i^{\sigma^*}(\theta_i) = U_i^\delta(\theta_i;\theta_i) = \sup_{\tilde{\theta}_i\in\Theta_i} U_i^\delta(\tilde{\theta}_i;\theta_i)$. Because v is C^1 on compact Θ, $|\partial u_i^\delta/\partial\theta_i|$ is uniformly bounded in $\tilde{\theta}_i$, so $U_i^\delta(\tilde{\theta}_i;\theta_i)$ is absolutely continuous on Θ_i. By the envelope theorem (Milgrom and Segal, 2002, Theorem 2), we then have

$$\frac{dU_i^{\sigma^*}(\theta_i)}{d\theta_i} = \left.\frac{\partial U_i^\delta(\tilde{\theta}_i;\theta_i)}{\partial\theta_i}\right|_{\tilde{\theta}_i=\theta_i}.$$

Moreover,

$$\begin{aligned}\frac{\partial U_i^\delta(\tilde{\theta}_i;\theta_i)}{\partial\theta_i} &= \int_{\Theta_j} \frac{\partial u_i^\delta}{\partial\theta_i}(\tilde{\theta}_i;(\theta_i, z))dF_j(z) \\ &= \int_{\Theta_j} \frac{\partial v}{\partial\theta_i}(\theta_i, z)[\pi^\delta(\tilde{\theta}_i, z)p_i + (1-\pi^\delta(\tilde{\theta}_i, z))x_i^\delta(\tilde{\theta}_i, z)]dF_j(z)\end{aligned}$$

by Assumption 2 and Leibniz's rule. Evaluating the expression at $\tilde{\theta}_i = \theta_i$ yields the stated result. The expression for an always-peaceful equilibrium follows directly by plugging in zero probability of war. □

A.6 Lemma 4

Lemma 4. *Suppose that Assumption 2 holds and fix an always-peaceful equilibrium σ^* of a crisis bargaining game Γ that has an incentive-compatible equivalent direct mechanism δ. If $x_i^\delta(\theta) \geq \bar{x}$ for* *a state i and all $\theta \in \Theta$, then the bounds*

$$V_i(\theta_i) - \underline{V}_i \geq U_i^{\sigma^*}(\theta_i) - U_i^{\sigma^*}(\underline{\theta}_i) \geq \bar{x}(V_i(\theta_i) - \underline{V}_i) \tag{A.6}$$

and

$$(1-\bar{x})(V_j(\theta_j) - \underline{V}_j) \geq U_j^{\sigma^*}(\theta_j) - U_j^{\sigma^*}(\underline{\theta}_j) \geq 0 \tag{A.7}$$

must hold for all $\theta_i \in \Theta_i$ and $\theta_j \in \Theta_j$.

Proof. Recall that for each state,

$$\frac{dU_i(\theta_i)}{d\theta_i} = \int_{\Theta_j} \frac{\partial v}{\partial \theta_i}(\theta_i, z) x_i^\delta(\theta_i, z) dF_j(z) \tag{A.8}$$

by Lemma 3. Fix a state i and opponent j such that $x_i^\delta(\theta) = 1 - x_j^\delta(\theta) \geq \bar{x}$ for all $\theta \in \Theta$.

Then, first consider equation (A.8) for state i. Because $1 \geq x_i^\delta(\theta) \geq \bar{x}$ for all θ and $\partial v/\partial\theta_i \geq 0$, the integrand can be bounded as

$$\frac{\partial v}{\partial \theta_i}(\theta_i, \theta_j) \geq \frac{\partial v}{\partial \theta_i}(\theta_i, \theta_j) x_i^\delta(\theta_i, \theta_j) \geq \frac{\partial v}{\partial \theta_i}(\theta_i, \theta_j)\bar{x}$$

for all $\theta_j \in \Theta_j$. Integrating both sides, we recover

$$\int_{\Theta_j} \frac{\partial v}{\partial \theta_i}(\theta_i, z) dF_j(z) \geq \frac{dU_i^{\sigma^*}(\theta_i)}{d\theta_i} \geq \bar{x} \int_{\Theta_j} \frac{\partial v}{\partial \theta_i}(\theta_i, z) dF_j(z).$$

By Assumption 2, Leibniz's rule, and the continuous differentiability of v, we can simplify the expression to $V_i'(\theta_i) \geq dU_i^{\sigma^*}(\theta_i)/d\theta_i \geq \bar{x}V_i'(\theta_i)$. Fixing θ_i and integrating with respect to Lebesgue measure from $\underline{\theta}_i$ to θ_i then yields

$$\int_{\underline{\theta}_i}^{\theta_i} V_i'(z)dz \geq \int_{\underline{\theta}_i}^{\theta_i} \frac{dU_i^{\sigma^*}}{dz}(z)dz \geq \bar{x} \int_{\underline{\theta}_i}^{\theta_i} V_i'(z)dz.$$

Because $U_i^{\sigma^*}$ and V_i are absolutely continuous, we have equation (A.6) by the fundamental theorem of calculus.

We derive bounds for opponent j by analogous argument. Because $1 - \bar{x} \geq x_j^\delta(\theta) \geq 0$, we have

$$\frac{\partial v}{\partial \theta_j}(\theta_i, \theta_j)(1-\bar{x}) \geq \frac{\partial v}{\partial \theta_j}(\theta_i, \theta_j) x_j^\delta(\theta_i, \theta_j) \geq 0$$

for all $\theta_i \in \Theta_i$, from which the identical steps as above imply equation (A.7). □

A.7 Lemma 5

Lemma 5. *Fix a direct mechanism δ such that $\pi^\delta(\theta) = 0$ for all $\theta \in \Theta$. If δ satisfies ex-post incentive compatibility and v is strictly increasing in both of its arguments, then $x^\delta(\theta_0, \theta_1)$ is non-decreasing in θ_0 and non-increasing in θ_1.*

If there is a two-sided indivisibility $(\underline{x}, \bar{x})$ and δ is robustly implementable by Definitions 6 and 7, *there exists a non-decreasing threshold $\theta_i^*(\theta_j)$ for each state i and any $\theta_j \in \Theta_j$ such that $x_i^\delta(\theta_i, \theta_j) \leq \underline{x}^{1-i}(1-\bar{x})^i$ for all $\theta_i < \theta_i^*(\theta_j)$ and $x_i^\delta(\theta_i, \theta_j) \geq \bar{x}^{1-i}(1-\underline{x})^i$ for all $\theta_i > \theta_i^*(\theta_j)$.*

Proof. Fix any θ_i, $\tilde{\theta}_i \in \Theta_i$ and $\theta_j \in \Theta_j$ such that $\theta_i > \tilde{\theta}_i$. By ex-post incentive compatibility, we know that

$$(v(\theta_i, \theta_j) - v(\tilde{\theta}_i, \theta_j))(x_i^\delta(\theta_i, \theta_j) - x_i^\delta(\tilde{\theta}_i, \theta_j)) \geq 0.$$

By v strictly increasing, it must be that x_i^δ is non-decreasing in θ_i and, by symmetric argument, non-increasing in θ_j.

Now suppose that there is a two-sided indivisibility and δ is robustly implementable. Then, for all θ, $\text{supp}(\delta(\theta)) = \{(0, x^\delta(\theta), t^\delta(\theta))\}$ and, by Definitions 6 and 7, $x^\delta(\theta) \notin (\underline{x}, \bar{x})$. Then, if for a fixed θ_j, $x_i^\delta(\theta) \leq \underline{x}^{1-i}(1-\bar{x})^i$ for all $\theta_i \in \Theta_i$, the statement holds with $\theta_i^*(\theta_j) = \bar{\theta}_i$.[3] Similarly, if $x_i^\delta(\theta) \geq \bar{x}^{1-i}(1-\underline{x})^i$ for all $\theta_i \in \Theta_i$, the statement holds with $\theta_i^*(\theta_j) = \underline{\theta}_i$.

If, however, there exist $\theta_i, \tilde{\theta}_i \in \Theta_i$ such that $x_i^\delta(\theta_i, \theta_j) \geq \bar{x}^{1-i}(1-\underline{x})^i > \underline{x}^{1-i}(1-\bar{x})^i \geq x_i^\delta(\tilde{\theta}_i, \theta_j)$ for a given θ_j, then we must show a unique threshold in state i's type. By fixing any θ_j and supposing that the above inequality holds for some types $\tilde{\theta}_i > \theta_i$, we arrive immediately at a contradiction to x_i^δ non-decreasing in θ_i.

Finally, i's threshold $\theta_i^*(\theta_j)$ must be non-decreasing in θ_j. To see this, fix any $\theta_j > \tilde{\theta}_j$. We have established that x_i^δ is non-increasing in θ_j and thus, for any $\theta_i \in \Theta_i$, we must have $x_i^\delta(\theta_i, \theta_j) \leq x_i^\delta(\theta_i, \tilde{\theta}_j)$. Therefore, $x_i^\delta(\theta_i, \theta_j) \geq \bar{x}^{1-i}(1-\underline{x})^i$ implies $x_i^\delta(\theta_i, \tilde{\theta}_j) \geq \bar{x}^{1-i}(1-\underline{x})^i$ for any $\theta_j > \tilde{\theta}_j$. In words, given a fixed θ_i, if the share induced by θ_j belongs to i's preferred side of the indivisibility, then so does the share induced by $\tilde{\theta}_j < \theta_j$. Now let $\Theta_i^*(\theta_j) \equiv \{\theta_i \in \Theta_i : x_i^\delta(\theta_i, \theta_j) \geq \bar{x}^{1-i}(1-\underline{x})^i\}$ be the set of state i's types that result in shares from the more preferable side of the indivisibility, given state j's type. Taking the infima, we have $\theta_i^*(\theta_j) = \inf \Theta_i^*(\theta_j)$ and, noting that monotonicity of x_i^δ in θ_j gives $\Theta_i^*(\theta_j) \subseteq \Theta_i^*(\tilde{\theta}_j)$ for all $\tilde{\theta}_j < \theta_j$, we have $\theta_i^*(\theta_j) \geq \theta_i^*(\tilde{\theta}_j)$. □

A.8 Lemma 6

Lemma 6. *Fix a direct mechanism δ such that $\pi^\delta(\theta) = 0$ for all $\theta \in \Theta$ and define the bargaining residual under truth-telling by $b(\theta_0, \theta_1) \equiv u_0^\delta(\theta_0; (\theta_0, \theta_1)) - v(\theta_0, \theta_1)p$. Suppose that δ satisfies ex-post incentive compatibility, v is strictly increasing in both of its arguments, and Θ_i is a compact interval for each state i. Then, b is absolutely continuous in each argument and*

$$\frac{\partial b}{\partial \theta_i}(\theta_i, \theta_j) = \frac{\partial v}{\partial \theta_i}(\theta_i, \theta_j)(x^\delta(\theta_i, \theta_j) - p)$$

for a state i and a.e. $\theta \in \Theta$.

Proof. Fix any two types $\theta_i, \tilde{\theta}_i \in \Theta_i$ such that $\theta_i > \tilde{\theta}_i$. By always-peace and ex-post incentive

[3] Recall that $x_0^\delta(\theta) \in [0, \underline{x}]$ implies $x_1^\delta(\theta) \in [1-\underline{x}, 1]$ and $x_0^\delta(\theta) \in [\bar{x}, 1]$ implies $x_1^\delta(\theta) \in [0, 1-\bar{x}]$.

compatibility, we require both

$$v(\theta_i,\theta_j)x_i^\delta(\theta_i,\theta_j)+t_i^\delta(\theta_i,\theta_j)\geq v(\theta_i,\theta_j)x_i^\delta(\tilde\theta_i,\theta_j)+t_i^\delta(\tilde\theta_i,\theta_j)$$
$$v(\tilde\theta_i,\theta_j)x_i^\delta(\tilde\theta_i,\theta_j)+t_i^\delta(\tilde\theta_i,\theta_j)\geq v(\tilde\theta_i,\theta_j)x_i^\delta(\theta_i,\theta_j)+t_i^\delta(\theta_i,\theta_j)$$

for all $\theta_j\in\Theta_j$. Combining these two expressions, we have

$$(v(\theta_i,\theta_j)-v(\tilde\theta_i,\theta_j))x_i^\delta(\theta_i,\theta_j)\geq u_i^\delta(\theta_i;(\theta_i,\theta_j))-u_i^\delta(\tilde\theta_i;(\tilde\theta_i,\theta_j))\geq(v(\theta_i,\theta_j)-v(\tilde\theta_i,\theta_j))x_i^\delta(\tilde\theta_i,\theta_j).$$

Subtracting $(v(\theta_i,\theta_j)-v(\tilde\theta_i,\theta_j))p_i$ from all terms, we then recover bounds on the difference of the bargaining residuals for any two types of state i given opponent j's type,

$$(v(\theta_i,\theta_j)-v(\tilde\theta_i,\theta_j))(x_i^\delta(\theta_i,\theta_j)-p_i)\geq b(\theta_i,\theta_j)-b(\tilde\theta_i,\theta_j)\geq(v(\theta_i,\theta_j)-v(\tilde\theta_i,\theta_j))(x_i^\delta(\tilde\theta_i,\theta_j)-p_i).$$

Dividing everything by $\theta_i-\tilde\theta_i$ and taking the limit as $\tilde\theta_i\to\theta_i$, we have, by Lemma 5 and the fact that v is C^1, convergence to

$$\frac{\partial b}{\partial\theta_i}(\theta_i,\theta_j)=\frac{\partial v}{\partial\theta_i}(\theta_i,\theta_j)(x_i^\delta(\theta_i,\theta_j)-p_i)$$

for almost every $\theta\in\Theta$. Setting $i=0$ establishes the claim for state 0.

To establish the condition for the opposing state $j=1$, note that the efficiency of peace implies that total payoffs in peace always equal the stakes, $u_0^\delta(\theta_0;(\theta_0,\theta_1))+u_1^\delta(\theta_1;(\theta_0,\theta_1))=v(\theta_0,\theta_1)$ for all $\theta\in\Theta$. Next, we can differentiate both sides with respect to θ_1. The right-hand side simply becomes $\partial v/\partial\theta_1$. For the left-hand side, we can put the formula just discovered to use,

$$\frac{\partial u_0^\delta}{\partial\theta_1}(\theta_0;(\theta_0,\theta_1))=\frac{\partial v}{\partial\theta_1}(\theta_0,\theta_1)-\frac{\partial v}{\partial\theta_1}(\theta_0,\theta_1)x_1^\delta(\theta_0,\theta_1)=\frac{\partial v}{\partial\theta_1}(\theta_0,\theta_1)x^\delta(\theta_0,\theta_1).$$

Finally, because $b(\theta_0,\theta_1)=u_0^\delta(\theta_0;(\theta_0,\theta_1))-v(\theta_0,\theta_1)p$, differentiating with respect to θ_1 recovers the stated expression for the opponent. □

A.9 Lemma 7

Lemma 7. *Fix an incentive-compatible direct mechanism δ such that $\pi^\delta(\theta)=0$ and $x_i^\delta(\theta)=1$ for some state i and for all $\theta\in\Theta$. Under Assumption 3, $T_i(\theta_i)\equiv\mathrm{E}_{\theta_j}[t_i^\delta(\theta)\,|\,\theta_i]$ is convex in θ_i.*

Proof. Take any incentive-compatible direct mechanism δ such that $\pi^\delta(\theta)=0$ and $x_i^\delta(\theta)=1$ for some state i and for all $\theta\in\Theta$. By Assumption 3, the conditional density of θ_j given θ_i is $f(\theta_j\,|\,\theta_i;\lambda)=(1+\lambda\theta_i\theta_j)/2$. Then, the interim expected transfer for state i with type θ_i reporting as type $\tilde\theta_i$ can be expressed

$$\mathrm{E}_{\theta_j}[t_i^\delta(\tilde\theta_i,\theta_j)\,|\,\theta_i]=\frac{1}{2}\int_{-1}^{1}t_i^\delta(\tilde\theta_i,z)(1+\lambda\theta_iz)dz=Y(\tilde\theta_i)+\lambda\theta_iZ(\tilde\theta_i)$$

where $Y(\theta_i) \equiv \frac{1}{2}\int_{-1}^{1} t_i^\delta(\theta_i, z)dz$ and $Z(\theta_i) \equiv \frac{1}{2}\int_{-1}^{1} z t_i^\delta(\theta_i, z)dz$. For every report $\tilde{\theta}_i \in \Theta_i$, interim expected transfers are affine in the true type θ_i.

State i's interim expected utility under δ when reporting $\tilde{\theta}_i$ is $V_i(\theta_i; \lambda) + Y(\tilde{\theta}_i) + \lambda\theta_i Z(\tilde{\theta}_i)$ by the antecedent assumptions on the direct mechanism δ. By incentive-compatibility, the true type θ_i maximizes this utility, hence

$$T_i(\theta_i) = \sup_{\tilde{\theta}_i \in [-1,1]} \left\{ Y(\tilde{\theta}_i) + \lambda\theta_i Z(\tilde{\theta}_i) \right\}.$$

A pointwise supremum of affine functions is convex, so T_i is convex on $[-1, 1]$. □

A.10 Lemma 8

Lemma 8. *Under Assumptions 3 and 4,* $\mathrm{E}v(\lambda)$ *is strictly increasing in* λ

Proof. By Assumptions 3 and 4,

$$\mathrm{E}v(\lambda) = \frac{1}{4}\int\int_{[-1,1]^2} \phi(z + \tilde{z})(1 + \lambda z\tilde{z})dzd\tilde{z} = \mathrm{E}v(0) + \lambda\Phi.$$

where

$$\Phi \equiv \frac{1}{4}\int\int_{[-1,1]^2} \phi(z + \tilde{z})z\tilde{z}dzd\tilde{z}. \tag{A.9}$$

To show that $\mathrm{E}v(\lambda)$ is strictly increasing in λ, it is sufficient to show $\Phi > 0$.

Now, the map $(\theta_0, \theta_1) \mapsto (-\theta_0, -\theta_1)$ is a bijection of $[-1, 1]^2$ with Jacobian determinant 1, fixing the product $\theta_0\theta_1$ while negating the sum $\theta_0 + \theta_1$. Applying the change of variables to Φ, we have

$$\Phi = \frac{1}{4}\int\int_{[-1,1]^2} \phi(-(z + \tilde{z}))z\tilde{z}dzd\tilde{z}. \tag{A.10}$$

If both expressions (A.9) and (A.10) equal Φ, so must their average,

$$\Phi = \frac{1}{4}\int\int_{[-1,1]^2} \varphi(z + \tilde{z})z\tilde{z}dzd\tilde{z} \tag{A.11}$$

where $\varphi(z) \equiv (\phi(z) + \phi(-z))/2$. By construction, φ is even and strictly convex as the average of two strictly convex functions. A strictly convex even function on $[-2, 2]$ has its unique minimum at zero and is strictly increasing on $[0, 2]$.

The integrand of equation (A.11) is now invariant under $(\theta_0, \theta_1) \mapsto (-\theta_0, -\theta_1)$, as evenness of φ absorbs the negation of the sum. The integral over $[-1, 1]^2$ therefore equals twice the integral over the half with $\theta_1 \geq 0$,

$$\Phi = \frac{1}{2}\int_0^1 \int_{-1}^1 \varphi(z + \tilde{z})z\tilde{z}dzd\tilde{z}.$$

Folding the integral over θ_0 at zero and applying $\theta_0 \mapsto -\theta_0$ to the integral over $\theta_0 \in [-1, 0]$,

$$\int_{-1}^{0} \varphi(z + \tilde{z}) z\tilde{z} dz = -\int_{0}^{1} \varphi(|z - \tilde{z}|) z\tilde{z} dz$$

by the evenness of φ. Combining with the integral over $\theta_0 \in [0, 1]$,

$$\Phi = \frac{1}{2} \int_{0}^{1} \int_{0}^{1} [\varphi(z + \tilde{z}) - \varphi(|z - \tilde{z}|)] \, z\tilde{z} dz d\tilde{z}. \tag{A.12}$$

On $(0, 1]^2$, $\theta_0 + \theta_1 - |\theta_0 - \theta_1| = 2 \min\{\theta_0, \theta_1\} > 0$ and hence $\theta_0 + \theta_1 > |\theta_0 - \theta_1|$. Because φ is strictly increasing on $[0, 2]$, $\varphi(\theta_0 + \theta_1) > \varphi(|\theta_0 - \theta_1|)$, so that the bracketed term in equation (A.12) is strictly positive. Together with $\theta_0\theta_1 > 0$ on $(0, 1]^2$, the integrand of (A.12) is strictly positive throughout $(0, 1]^2$, a set of full Lebesgue measure in $[0, 1]^2$. A non-negative measurable function that is strictly positive on a full-measure set has a strictly positive integral, concluding the proof. □

A.11 Lemma 9

Lemma 9. *Suppose Assumptions 3 and 4 hold. If* $\lambda = 0$ *and*

$$c_0 + c_1 \geq p_j(V_j(1; 0) - V_i(-1; 0)) \tag{A.13}$$

for a state i *and opponent* j*, then there exists a transfer-protocol crisis bargaining game that admits an always-peaceful equilibrium.*

Proof. For proof by example, consider the simultaneous-move fixed-transfer game described in the section on the transfer-protocol subclass and illustrated in Figure 3.

Fix a state i and type θ_j. If peace is reached, opponent j receives a fixed payoff of t. If war, j's expected payoff is $V_j(\theta_j; 0)p_j - c_j$, recalling that $\lambda = 0$. Therefore, opponent j prefers to settle if and only if $t \geq V_j(\theta_j; 0)p_j - c_j$. Because $V_j(\theta_j; 0)$ is increasing in θ_j, a sufficient condition for j to prefer settling over war is given by

$$t \geq V_j(1; 0)p_j - c_j. \tag{A.14}$$

Now, fix a type θ_i. If peace is reached, state i receives a payoff of $V_i(\theta_i; 0) - t$. On the other hand, war yields a payoff of $V_i(\theta_i; 0)p_i - c_i$ to state i. Then, a type θ_i prefers settlement to war if and only if $V_i(\theta_i; 0) - t \geq V_i(\theta_i; 0)p_i - c_i$, or equivalently $t \leq V_i(\theta_i; 0)p_j + c_i$. Because $V_i(\theta_i; 0)$ is increasing in θ_i, a sufficient condition is

$$t \leq V_i(-1; 0)p_j + c_i. \tag{A.15}$$

Together, by inequalities (A.14) and (A.15), settlement is preferred to war by both states if and only if $V_j(1; 0)p_j - c_j \leq t \leq V_i(-1; 0)p_j + c_i$. It is straightforward to see that this interval is non-empty if and only if inequality (A.13) holds. Then, choosing such a t yields the result. □

A.12 Proposition 5

Proposition 5. *Suppose Assumptions 3 and 4 hold and*

$$c_0 + c_1 < p_j(\mathrm{E}v(\lambda) - V_i(0;\lambda)) \tag{A.16}$$

for a state i and opponent j. Then, there does not exist an incentive-compatible direct mechanism δ such that $\pi = 0$ and $x_i = 1$ for all $(\pi, x, t) \in \operatorname{supp}(\delta(\theta))$ and all $\theta \in \Theta$. Therefore, no such game of the transfer-protocol subclass admits an always-peaceful equilibrium.

Proof. Take any transfer-protocol crisis bargaining game Γ. Because correlation in this density operates through the product $\theta_0\theta_1$, the interim expected stakes for a type $\theta_i = 0$ is independent of λ. Using $v(0, \theta_j) = \phi(\theta_j)$, we have

$$V_i(0;\lambda) = \int_{-1}^{1} v(0, z)dF(z \mid 0; \lambda) = \frac{1}{2}\int_{-1}^{1} \phi(z)dz.$$

For contradiction, suppose there exists an incentive-compatible direct mechanism δ such that $\pi^\delta(\theta) = 0$ and $x_i^\delta(\theta) = 1$ for all $\theta \in \Theta$, and that inequality (A.16) holds. Also recall that in the transfer-protocol subclass, always-peace requires $T_i(\theta_i) = \mathrm{E}_{\theta_j}[t_i^\delta(\theta) \mid \theta_i] \leq 0$.

By Lemma 7, T_i is convex on $[-1, 1]$. By the VA criterion (Assumption 1), state i must weakly prefer their expected payoff from peace to that from war at the interim stage, $V_i(\theta_i;\lambda) + T_i(\theta_i) \geq V_i(\theta_i;\lambda)p_i - c_i$. Rearranging and evaluating at $\theta_i = 0$, noting $V_i(0;\lambda) = V_i(0)$, yields the condition $T_i(0) \geq -V_i(0)p_j - c_i$.

By Assumption 3, the marginal of θ_i is uniform on $[-1, 1]$, so $\mathrm{E}\theta_i = 0$. Since T_i is convex, $\mathrm{E}T_i(\theta_i) \geq T_i(\mathrm{E}\theta_i) = T_i(0)$ by Jensen's inequality. Together, these two expressions imply that

$$\mathrm{E}T_i(\theta_i) \geq -V_i(0)p_j - c_i. \tag{A.17}$$

Again by the VA criterion (Assumption 1), state j's interim payoff must satisfy $T_j(\theta_j) \geq V_j(\theta_j;\lambda)p_j - c_j$ for all $\theta_j \in \Theta_j$. Taking expectations over θ_j uniformly distributed on $[-1, 1]$, we have $\mathrm{E}T_j(\theta_j) \geq p_j\mathrm{E}V_j(\theta_j;\lambda) - c_j$. By definition of V_j and the law of iterated expectations,

$$\mathrm{E}V_j(\theta_j;\lambda) = \int\int_{[-1,1]} v(z, \tilde{z}) f(z, \tilde{z}; \lambda) dz d\tilde{z} = \mathrm{E}v(\lambda).$$

Plugging into j's constraint yields

$$\mathrm{E}T_j(\theta_j) \geq p_j\mathrm{E}v(\lambda) - c_j. \tag{A.18}$$

Now, recall that transfers are efficient, and thus $\mathrm{E}T_i(\theta_i) + \mathrm{E}T_j(\theta_j) = 0$. Then, inequalities (A.17) and (A.18) together imply

$$0 = \mathrm{E}T_i + \mathrm{E}T_j \geq p_j(\mathrm{E}v(\lambda) - V_i(0)) - (c_0 + c_1),$$

a contradiction. $\square$

B Proofs

This section presents the proofs for formal statements presented in the main text. I provide a restatement of the result for convenience.

B.1 Proposition 1

Proposition 1. *Suppose power p and resolve (c_0, c_1) are common knowledge, but there are uncertain stakes v. Then, there exists an incentive-compatible direct mechanism δ such that, for all $\theta \in \Theta$, $\pi = 0$ and $t = 0$ for all $(\pi, x, t) \in \operatorname{supp}(\delta(\theta))$. Therefore, there exists a share-protocol crisis bargaining game that admits an always-peaceful equilibrium.*

Proof. Consider the direct mechanism δ defined by $\delta(\theta)(\{(0, p, 0)\}) = 1$ for all $\theta \in \Theta$. Fix a state i, true type θ_i, and report $\tilde{\theta}_i$. For every $\theta_j \in \Theta_j$, $u_i^\delta(\tilde{\theta}_i; (\theta_i, \theta_j)) = v(\theta_i, \theta_j)p_i$ and hence $U_i^\delta(\tilde{\theta}_i; \theta_i) = \mathrm{E}_{\theta_j}[v(\theta_i, \theta_j)p_i \mid \theta_i] = V_i(\theta_i)p_i$, which does not vary in the report $\tilde{\theta}_i$ and hence satisfies the incentive-compatibility constraint. Moreover, because $c_i > 0$ for each state i, it is clear to see the interim participation constraint is also satisfied, $U_i^\delta(\theta_i; \theta_i) > V_i(\theta_i)p_i - c_i$.

By Lemma 1, there exists a crisis bargaining game with a Bayesian Nash equilibrium of which δ is an equivalent direct mechanism. Because $t = 0$ for all $(\pi, x, t) \in \operatorname{supp}(\delta(\theta))$ for all $\theta \in \Theta$, and choosing off-path transfers such that $t = 0$, every $a \in A$ satisfies $t = 0$ for all $(\pi, x, t) \in \operatorname{supp}(\gamma(a))$. Therefore, the constructed game belongs to the share-protocol subclass. $\square$

B.2 Theorem 1

Theorem 1. *Suppose power p and resolve (c_0, c_1) are common knowledge, but there are uncertain stakes v. If Assumption 2 holds and there is a one-sided indivisibility with $\bar{x} \in [0, 1]$ such that*

$$c_0 + c_1 < (\bar{x} - p_i)(\bar{V}_j - \underline{V}_i) - (1 - \bar{x})(\mathrm{E}v - \underline{V}_j) \tag{2}$$

for a state i and opponent j, then there does not exist an incentive-compatible direct mechanism δ such that, for all $\theta \in \Theta$, $\pi = 0$ and $x_i \geq \bar{x}$ for all $(\pi, x, t) \in \operatorname{supp}(\delta(\theta))$. Therefore, there does not exist a crisis bargaining game with such a one-sided indivisibility that admits an always-peaceful equilibrium.

Proof. Fix a state i, opponent j, and equilibrium σ^*. Suppose for contradiction that inequality (2) holds and there exists an incentive-compatible direct mechanism δ such that, for all $\theta \in \Theta$, $\pi = 0$ and $x_i \geq \bar{x}$ for all $(\pi, x, t) \in \operatorname{supp}(\delta(\theta))$. Since every realized outcome in the support of $\delta(\theta)$ satisfies $x_i \geq \bar{x}$, the induced expected peaceful share also satisfies $x_i^\delta(\theta) \geq \bar{x}$ for all $\theta \in \Theta$.

First, we can place a lower bound on $\mathrm{E}U_j^{\sigma^*} \equiv \mathrm{E}_{\theta_j} U_j^{\sigma^*}(\theta_j)$. By Lemma 2 and the definition of equivalent direct mechanism, $U_j^{\sigma^*}(\bar{\theta}_j) \geq \bar{V}_j p_j - c_j$. Additionally, by Lemma 4, we know that

$U_j^{\sigma^*}(\bar{\theta}_j) - U_j^{\sigma^*}(\underline{\theta}_j) \leq (1-\bar{x})(\bar{V}_j - \underline{V}_j)$. Combining the two inequalities and rearranging,

$$\begin{aligned} U_j^{\sigma^*}(\underline{\theta}_j) &\geq U_j^{\sigma^*}(\bar{\theta}_j) - (1-\bar{x})(\bar{V}_j - \underline{V}_j) \\ &\geq \bar{V}_j p_j - c_j - (1-\bar{x})(\bar{V}_j - \underline{V}_j) = (\bar{x} - p_i)\bar{V}_j + (1-\bar{x})\underline{V}_j - c_j. \end{aligned} \tag{A.19}$$

By Lemma 4, $U_j^{\sigma^*}(\theta_j) \geq U_j^{\sigma^*}(\underline{\theta}_j)$ for all $\theta_j \in \Theta_j$, and hence $\mathrm{E}U_j^{\sigma^*} \geq (\bar{x} - p_i)\bar{V}_j + (1-\bar{x})\underline{V}_j - c_j$.

Next, we can establish an upper bound on $\mathrm{E}U_j^{\sigma^*}$. By Lemma 4, we have that $U_i^{\sigma^*}(\theta_i) \geq U_i^{\sigma^*}(\underline{\theta}_i) + \bar{x}(V_i(\theta_i) - \underline{V}_i)$ for all $\theta_i \in \Theta_i$. Taking expectations over $\theta_i \sim F_i$, we can then recover $\mathrm{E}U_i^{\sigma^*} \geq U_i(\underline{\theta}_i) + \bar{x}(\mathrm{E}_{\theta_i} V_i(\theta_i) - \underline{V}_i)$. By Assumption 2 and Lemma 2, we have $\mathrm{E}U_i^{\sigma^*} \geq \bar{x}\mathrm{E}v - (\bar{x} - p_i)\underline{V}_i - c_i$. Because $\mathrm{E}U_i^{\sigma^*} = \mathrm{E}v - \mathrm{E}U_j^{\sigma^*}$ for any always-peaceful equilibrium σ^*, we can deduce that

$$\mathrm{E}U_j^{\sigma^*} = \mathrm{E}v - \mathrm{E}U_i^{\sigma^*} \leq (1-\bar{x})\mathrm{E}v + (\bar{x} - p_i)\underline{V}_i + c_i. \tag{A.20}$$

Together, bounds (A.19) and (A.20) imply $c_i + c_j \geq (\bar{x} - p_i)(\bar{V}_j - \underline{V}_i) - (1-\bar{x})(\mathrm{E}v - \underline{V}_j)$. □

B.3 Proposition 2

Proposition 2. *Suppose power p and resolve (c_0, c_1) are common knowledge, but there are uncertain stakes v. If Assumption 2 holds and*

$$c_0 + c_1 < p_j(\bar{V}_j - \underline{V}_i)$$

for a state i and opponent j, then there does not exist an incentive-compatible direct mechanism δ such that, for all $\theta \in \Theta$, $\pi = 0$ and $x_i = 1$ for all $(\pi, x, t) \in \mathrm{supp}(\delta(\theta))$. Therefore, there does not exist a transfer-protocol crisis bargaining game that admits an always-peaceful equilibrium.

Proof. Setting $\bar{x} = 1$, the result follows immediately by Theorem 1. □

B.4 Proposition 3

Proposition 3. *Suppose power p and resolve (c_0, c_1) are commonly known, but there is uncertainty about stakes v. If there is a two-sided indivisibility given by any $\underline{x} \in [0, 1)$ and $\bar{x} \in (\underline{x}, 1]$, then there exists an incentive-compatible direct mechanism δ such that $\pi = 0$ for all $(\pi, x, t) \in \mathrm{supp}(\delta(\theta))$, for all $\theta \in \Theta$. Therefore, there exists such a crisis bargaining game with such a two-sided indivisibility that admits an always-peaceful equilibrium.*

Proof. First, for any two-sided indivisibility such that $p \notin (\underline{x}, \bar{x})$, then a direct mechanism δ such that $\delta(\theta)(\{(0, p, 0)\}) = 1$ for all $\theta \in \Theta$ has already been shown to admit an always-peaceful equilibrium by Proposition 1. Therefore, suppose $p \in (\underline{x}, \bar{x})$.

For proof by example, conjecture the following direct mechanism. For any reported types $\tilde{\theta}$, the mechanism designer assigns the outcome

$$\delta(\tilde{\theta}) = \begin{cases} (0, \bar{x}, -v(\tilde{\theta})(\bar{x} - p)) & \text{with prob. } q \\ (0, \underline{x}, v(\tilde{\theta})(p - \underline{x})) & \text{with prob. } 1 - q. \end{cases}$$

State 0 with type θ_0 then has an interim expected utility for reporting as type $\tilde{\theta}_0$ of

$$U_0^\delta(\tilde{\theta}_0; \theta_0) = V_0(\theta_0)(q\bar{x} + (1-q)\underline{x}) - \mathrm{E}_{\theta_1}[v_0(\tilde{\theta}_0, \theta_1) \mid \theta_0](q(\bar{x} - p) - (1-q)(p - \underline{x})). \tag{A.21}$$

Further, let $q = (p - \underline{x})/(\bar{x} - \underline{x})$ so that $q\bar{x} + (1-q)\underline{x} = p$ and $q(\bar{x} - p) - (1-q)(p - \underline{x}) = 0$. Then, the expression (A.21) reduces to $U_0^\delta(\tilde{\theta}_0; \theta_0) = V_0(\theta_0)p_0$, independent of the report. First, the direct mechanism satisfies the interim participation constraint, as $V_0(\theta_0)p_0 \geq V_0(\theta_0)p_0 - c_0$. Second, the direct mechanism trivially satisfies the incentive-compatibility constraint, as the interim expected utility does not depend on the reported type.

Similarly, the interim expected utility for state 1 of type θ_1 reporting as type $\tilde{\theta}_1$ simplifies to $U_1^\delta(\tilde{\theta}_1; \theta_1) = V_1(\theta_1)p_1$ and hence an analogous argument follows for state 1.

By Lemma 1, there exists a crisis bargaining game with a Bayesian Nash equilibrium of which δ is an equivalent direct mechanism. Because $x \notin (\underline{x}, \bar{x})$ for all $(\pi, x, t) \in \mathrm{supp}(\delta(\theta))$ for all $\theta \in \Theta$, and choosing off-path settlements so that $x \notin (\underline{x}, \bar{x})$ without loss of generality, every $a \in A$ satisfies $x \notin (\underline{x}, \bar{x})$ for all $(\pi, x, t) \in \mathrm{supp}(\gamma(a))$. Therefore, the constructed game has the two-sided indivisibility given by $(\underline{x}, \bar{x})$. □

B.5 Theorem 2

Theorem 2. *Suppose power p and resolve (c_0, c_1) are common knowledge, but there are uncertain stakes v. Further, let v be strictly increasing in both of its arguments and Θ_i be a compact interval for each state i. Then, if there is a two-sided indivisibility given by $\underline{x} \in [0, 1)$ and $\bar{x} \in (\underline{x}, 1]$ such that*

$$c_0 + c_1 < \frac{(\bar{x} - p)(p - \underline{x})}{\bar{x} - \underline{x}} \max_i \left\{ \max_{\theta_j} [v(\bar{\theta}_i, \theta_j) - v(\underline{\theta}_i, \theta_j)] \right\}, \tag{3}$$

there does not exist a robustly implementable direct mechanism δ such that $\pi = 0$ for all $(\pi, x, t) \in$ $\mathrm{supp}(\delta(\theta))$, for all $\theta \in \Theta$. Therefore, there does not exist a crisis bargaining game with such a two-sided indivisibility that admits a robust always-peaceful equilibrium.

Proof. Suppose, for contradiction, there exists such a robustly implementable direct mechanism δ. Recall b as the bargaining residual defined by Lemma 6. Ex-post voluntary agreements requires $u_i^\delta(\theta_i; (\theta_i, \theta_j)) \geq v(\theta_i, \theta_j)p_i - c_i$ for each state i. Combining these, we recover a bound on the bargaining residual, $c_1 \geq b(\theta) \geq -c_0$ for all $\theta \in \Theta$.

By Lemma 5, there is a threshold $\theta_i^*(\theta_j) \in \Theta_i$ that partitions state i's types into settlements with shares below $\underline{x}$ and above $\bar{x}$. Further, let $v_i^*(\theta_j) \equiv v(\theta_i^*(\theta_j), \theta_j)$ be the value of the stakes at state i's threshold and $b_i^*(\theta_j) \equiv b(\theta_i^*(\theta_j), \theta_j)$ the bargaining residual at state i's threshold, given any opponent type θ_j.

Now fix $\theta_1 \in \Theta_1$. Then, by Lemma 6,

$$\frac{\partial b}{\partial \theta_0}(\theta_0, \theta_1) = \frac{\partial v}{\partial \theta_0}(\theta_0, \theta_1)(x^\delta(\theta_0, \theta_1) - p) \le -\frac{\partial v}{\partial \theta_0}(\theta_0, \theta_1)(p - \underline{x})$$

for any $\theta_0 < \theta_0^*(\theta_1)$. Because b is absolutely continuous, by integrating from $\underline{\theta}_0$ to $\theta_0^*(\theta_1)$, we have

$$b(\underline{\theta}_0, \theta_1) - b_0^*(\theta_1) \ge (p - \underline{x})(v_0^*(\theta_1) - v(\underline{\theta}_0, \theta_1)).$$

Likewise, we also have

$$\frac{\partial b}{\partial \theta_0}(\theta_0, \theta_1) = \frac{\partial v}{\partial \theta_0}(\theta_0, \theta_1)(x^\delta(\theta_0, \theta_1) - p) \ge \frac{\partial v}{\partial \theta_0}(\theta_0, \theta_1)(\bar{x} - p) \tag{A.22}$$

by Lemma 6, for any $\theta_0 > \theta_0^*(\theta_1)$. Integrating again, this time from $\theta_0^*(\theta_1)$ to $\bar{\theta}_0$, yields

$$b(\underline{\theta}_0, \theta_1) - b_0^*(\theta_1) \ge (\underline{x} - p)(v(\bar{\theta}_0, \theta_1) - v_0^*(\theta_1)). \tag{A.23}$$

Through inequalities (A.22) and (A.23) and the bound on the residual b, we establish a bound on the stakes of the dispute at state 0's threshold,

$$v(\underline{\theta}_0, \theta_1) + \frac{c_0 + c_1}{p - \underline{x}} \ge v_0^*(\theta_1) \ge v(\bar{\theta}_0, \theta_1) - \frac{c_0 + c_1}{\bar{x} - p}$$

for all types $\theta_1 \in \Theta_1$. Rearranging, we require that

$$c_0 + c_1 \ge \frac{(\bar{x} - p)(p - \underline{x})}{\bar{x} - \underline{x}}(v(\bar{\theta}_0, \theta_1) - v(\underline{\theta}_0, \theta_1)) \tag{A.24}$$

for every $\theta_1 \in \Theta_1$.

State 1 follows by symmetric argument. Fixing $\theta_0 \in \Theta_0$, we can again apply Lemma 6 to bound the partial derivative of b,

$$\frac{\partial b}{\partial \theta_1}(\theta_0, \theta_1) \le -\frac{\partial v}{\partial \theta_1}(p - \underline{x}); \qquad \theta_1 > \theta_1^*(\theta_0) \tag{A.25}$$

$$\frac{\partial b}{\partial \theta_1}(\theta_0, \theta_1) \ge \frac{\partial v}{\partial \theta_1}(\theta_0, \theta_1)(\bar{x} - p); \qquad \theta_1 < \theta_1^*(\theta_0). \tag{A.26}$$

Integrating inequality (A.25) from $\theta_1^*(\theta_0)$ to $\bar{\theta}_1$ and inequality (A.26) from $\underline{\theta}_1$ to $\theta_1^*(\theta_0)$ in the same way recovers, respectively,

$$b_1^*(\theta_0) - b(\theta_0, \bar{\theta}_1) \ge (p - \underline{x})(v(\theta_0, \bar{\theta}_1) - v_1^*(\theta_0)) \tag{A.27}$$

$$b_1^*(\theta_0) - b(\theta_0, \underline{\theta}_1) \ge (\bar{x} - p)(v_1^*(\theta_0) - v(\theta_0, \underline{\theta}_1)), \tag{A.28}$$

Together with the residual bound, inequalities (A.27) and (A.28) imply

$$c_0 + c_1 \geq \frac{(\bar{x} - p)(p - \underline{x})}{\bar{x} - \underline{x}}(v(\theta_0, \bar{\theta}_1) - v(\theta_0, \underline{\theta}_1)) \tag{A.29}$$

for every $\theta_0 \in \Theta_0$.

If both (A.24) and (A.29) hold for every $\theta_0 \in \Theta_0$ and $\theta_1 \in \Theta_1$, respectively, then we have a contradiction to inequality (3). □

B.6 Proposition 4

Proposition 4. *Suppose Assumptions 3 and 4 hold, and consider transfer-protocol crisis bargaining games with uncertain stakes. If* $V_j(1;0) - V_i(-1;0) < \mathrm{E}v(1) - V_i(0;1)$ *for a state* i *and opponent* j*, then there exist primitives* $(p, c_0, c_1) \in (0,1) \times \mathbb{R}_+^2$ *and a unique* $\bar{\lambda} \in (0,1)$ *such that no game admits always-peaceful equilibria under any* $\lambda > \bar{\lambda}$ *but there do exist games that admit always-peaceful equilibria under* $\lambda = 0$*.*

Proof. Fix a state i and opponent j, and define $M(\lambda) \equiv \mathrm{E}v(\lambda) - V_i(0;\lambda)$. By Assumption 3, $f_j(\theta_j \,|\, 0; \lambda) = 1/2$ for every $\lambda \in [0,1]$ and hence

$$V_i(0;\lambda) = \frac{1}{2}\int_{-1}^{1} \phi(z)dz \equiv \hat{V}_i(0),$$

which is constant in λ. Therefore, $M(\lambda) = \mathrm{E}v(\lambda) - \hat{V}_i(0)$. By Lemma 8, M is continuous and strictly increasing.

Next, by Assumption 3, both states i and j of any type $\theta_k \in [-1,1]$ have the same interim expected stakes at $\lambda = 0$,

$$V_i(\theta_k;0) = V_j(\theta_k;0) = \frac{1}{2}\int_{-1}^{1} \phi(\theta_k + z)dz \equiv m(\theta_k).$$

By Assumption 4, we can deduce m is strictly increasing. Then, by the law of iterated expectations,

$$\mathrm{E}v(0) = \mathrm{E}_{\theta_k}[V_k(\theta_k;0)] = \mathrm{E}_{\theta_k}[m(\theta_k)] = \frac{1}{2}\int_{-1}^{1} m(z)dz.$$

Because m is strictly increasing, its average over $[-1,1]$ is strictly less than its maximum, and hence $\mathrm{E}v(0) < m(1)$. Therefore, $M(0) = \mathrm{E}v(0) - m(0) < m(1) - m(0) < m(1) - m(-1)$. Using the fact that $m(1) = V_k(1;0)$ and $m(-1) = V_k(-1;0)$ for either $k \in \{i,j\}$, we can conclude that $M(0) < V_j(1;0) - V_i(-1;0) < M(1)$, where the latter inequality follows by the antecedent.

We now establish a threshold. First, choose any $\alpha \in (V_j(1;0) - V_i(-1;0), M(1))$. Because M is continuous and strictly increasing, there exists a unique $\bar{\lambda} \in (0,1)$ such that $M(\bar{\lambda}) = \alpha$ by the intermediate value theorem.

Then, choose primitives (p, c_0, c_1) so that the Proposition 5 impossibility condition binds at exactly $\bar{\lambda}$. Always-peace is impossible when $c_0 + c_1 < p_j M(\lambda)$, so it is enough to choose primitives

(p, c_0, c_1) such that $c_0 + c_1 = p_j M(\bar{\lambda})$. With this choice, it is impossible to construct a game that produces always-peace for any $\lambda > \bar{\lambda}$.

To see these primitives exist and can be selected, choose a $p \in (0,1)$ such that $p_j\alpha < p_i\phi(-2)$. Given this, select $c_j = p_j \min\{\alpha, \phi(-2)\}/2$ and $c_i \equiv p_j\alpha - c_j$. Now we can verify that $c_i + c_j = p_j\alpha$, so the threshold of Proposition 5 binds at $\bar{\lambda}$. Moreover, this guarantees our conditions on the minimum stakes are satisfied, $\phi(-2) > \max_k c_k/p_k$, as required for ϕ by Assumption 4.

Finally, holding these primitives (p, c_0, c_1) fixed, $c_0 + c_1 = p_j\alpha > p_j(V_j(1;0) - V_i(-1;0))$ by construction. Therefore, by Lemma 9, there exists a transfer-protocol crisis bargaining game that admits an always-peaceful equilibrium at $\lambda = 0$. □

Appendix References

Milgrom, Paul and Ilya Segal. 2002. "Envelope Theorems for Arbitrary Choice Sets." *Econometrica* 70(2):583–601.