Online Appendix for "Competitive Diplomacy in Bargaining and War"

Contents

A Additional Model Details and Discussion

A.1 State Variables and Transitions

There are three state variables. The first simply traces the previous period's distribution of the pie, $z_t = o_{t-1} \in \mathbb{R}$ for each period $t \geq 2$ where o_t gives the outcome in period t and without loss of generality $z_1 = 0$. The outcome of a period corresponds to the interaction's resulting division of the pie. In particular, period t's outcome is given by $o_t = 1$ if country 1 wins a war, $o_t = 0$ if country 2 wins a war, and $o_t = x$ for a peace deal reached at $x \in \mathbb{R}$.

Second, the state variable $s_t \in S$ provides the measure of country 1's relative strength, where S is a finite subset of $(0, 1)$. Strengths may evolve flexibly over time according to a Markov transition function $q : S \times A \to S$, where $a_t \in A \equiv \{0, 1\}^2$ are the war actions taken in period t.

Finally, the state variable b_t denotes the crisis bargaining status of the game in period t. In particular, $b_t = 0$ is an inactive state where an outcome persists and each country consumes their status quo share of the pie, z_t. On the other hand, $b_t = 1$ implies countries have an opportunity to engage in diplomatic competition and crisis bargaining.

A key feature of the model is that these outcomes can persist into future periods. Specifically, the game begins in bargaining $b_t = 1$ and, at the end of every period, the state variable b_t transitions to $b_{t+1} = 0$ with probability $\theta \in [0, 1]$ when the current division is a war outcome and with probability $\lambda \in [0, 1]$ when the division is the result of a peace deal.

Note that a strategy σ_i does not take b_t or z_t as input because (1) when $b_t = 0$, countries simply consume their share of the pie according to the previous period's division, which is equal to the current state variable z_t, and (2) given $b_t = 1$, state variable z_t is no longer payoff relevant. Thus, I refer to state $(1, z_t, s_t)$ as state s_t when it is implied that $b_t = 1$.

A.2 Budget Constraints

Budget constraints are given by $\mathcal{X}_1 := \{x \in \mathbb{R} : x \geq X_1\}$ and $\mathcal{X}_2 := \{x \in \mathbb{R} : x \leq X_2\}$ where X_1 and X_2 are the greatest credible agreements for country 1 and 2, respectively. This is consistent with the standard approach in dynamic crisis bargaining, where a lack of liquidity prevents side payments that can placate an opponent. Most typically, $X_1 = 0$ and $X_2 = 1$. The main text focuses on $X_1 = -\infty$ and $X_2 = \infty$ to avoid preventive wars and isolate the mechanisms of interest.

A.3 Payoffs

After the outbreak of war, country i wins a pie of size 1 today with probability s_i and tomorrow (discounted at rate δ) they either get, with probability θ, $Z_i^{war}(s)$, which is the ex-ante expected future value of the war outcome persisting (i.e., prior to knowledge over whether i or i's opponent wins the war), or with complementary probability $1-\theta$ they return to bargaining under a new state s'. Given the state $s \in S$, each country i has an expected war value of

$$W_i(s) = s_i - c_i(s) + \delta \left[\theta Z_i^{war}(s) + (1-\theta) \sum_{s' \in S} V_i(s') q(s'|s, a_W) \right] \tag{A1}$$

where

$$Z_i^{war}(s) = s_i + \delta \left[\theta Z_i^{war}(s) + (1-\theta) \sum_{s' \in S} V_i(s') q(s'|s, a_W) \right]. \tag{A2}$$

and a_W denotes actions that involve countries choosing to fight a war. Using equation (A2) to solve for $Z_i^{war}(s)$, equation (A1) simplifies to equation (2) in the main text.

If countries succeed in cooperating, let the expected settlement be given by $N(s)$. In equilibrium, this expectation will correspond to the Nash bargaining solution due to symmetric effort strategies. Then, countries receive a payoff according to the expected settlement today, less the amount they expect to exert according to their mixed diplomatic effort strategy. Tomorrow, countries receive payoffs according to the same settlement allocation with probability λ and return to the bargaining table under a new state s' with probability $1-\lambda$. Formally, we can write

$$U_i(s) = u_i(N(s)) - \int e dF_s^*(e) + \delta \left[\lambda Z_i^{peace}(s) + (1-\lambda) \sum_{s' \in S} V_i(s') q(s'|s, a_U) \right] \tag{A3}$$

where

$$Z_i^{peace}(s) = u_i(N(s)) + \delta \left[\lambda Z_i^{peace}(s) + (1-\lambda) \sum_{s' \in S} V_i(s') q(s'|s, a_U) \right]. \tag{A4}$$

and a_U denotes actions that involve countries choosing to cooperate. As before, it is necessary to account for the uncertainty over how long peace will persist with $Z_i^{peace}(s)$. Using equation (A4) to solve for $Z_i^{peace}(s)$, equation (A3) simplifies to equation (3) in the main text.

A.4 Equilibrium Offers

When bargaining surplus exists, a country will offer the least generous settlement that facilitates peace—that is, they will offer the point that makes their opponent indifferent or their opponent's greatest credible settlement. First, consider the case without budget constraints. Since diplomatic effort is a sunk cost at the point at which a country makes a proposal, an agenda setter $-i$ will offer an $\bar{x}_i(s)$ that solves $U_i(\bar{x}_i(s); s) = W_i(s)$. In particular, we can solve for values that make countries 1 and 2 indifferent in state s,

$$W_i(s) = u_i(x) + \frac{\delta}{1-\delta\lambda}\left[\lambda u_i(x) + (1-\lambda)\sum_{s'\in S} V_i(s')q(s'|s, a_U)\right]. \tag{A5}$$

Solving (A5) for $u_i(x)$, we get

$$u_i(x) = (1-\delta\lambda)W_i(s) - \delta(1-\lambda)\sum_{s'\in S} V_i(s')q(s'|s, a_U),$$

which, after plugging in $u_1(x) = x$ and $u_2(x) = 1 - x$, yields implicit solutions,

$$\bar{x}_1(s) = (1-\delta\lambda)W_1(s) - \delta(1-\lambda)\sum_{s'\in S} V_1(s')q(s'|s, a_U) \tag{A6}$$

$$\bar{x}_2(s) = 1 - (1-\delta\lambda)W_2(s) + \delta(1-\lambda)\sum_{s'\in S} V_2(s')q(s'|s, a_U). \tag{A7}$$

The relationship between these indifference points and the indifference points of the standard static bargaining model of war is apparent. In particular, perfectly impatient countries are indifferent at the same settlement as countries in the standard bargaining model of war with complete and perfect information. To see this, consider when $\delta = 0$ so that equation (A6) becomes $\bar{x}_1(s) = s - c_1$ and equation (A7) becomes $\bar{x}_2(s) = s + c_2$. These are equivalent to the standard static bargaining range from the literature.

Upon making a proposal, countries will choose an offer that allocates themselves the largest quantity that their opponent would accept to avoid war. The decision to extend and accept an offer occurs after both countries have already incurred diplomatic effort. This is going to be equal to the adversary's indifference deal unless budget constraints get in the way, in which case an agenda setter will extract the greatest credible settlement. If the greatest credible settlement is worse than the agenda setter's indifference deal, however, they will prefer war to cooperation. Then, we can formally define the equilibrium offers as

$$x_1^*(s) = \begin{cases} \bar{x}_2(s) & \text{if } X_2 \geq \bar{x}_2(s) \geq \bar{x}_1(s) \\ X_2 & \text{if } \bar{x}_2(s) > X_2 \geq \bar{x}_1(s) \\ x & \text{for any } x \in \mathcal{R}_2(s) \text{ otherwise} \end{cases} \tag{A8}$$

$$x_2^*(s) = \begin{cases} \bar{x}_1(s) & \text{if } \bar{x}_2(s) \geq \bar{x}_1(s) \geq X_1 \\ X_1 & \text{if } \bar{x}_2(s) \geq X_1 > \bar{x}_1(s) \\ x & \text{for any } x \in \mathcal{R}_1(s) \text{ otherwise.} \end{cases} \tag{A9}$$

where $\mathcal{R}_1(s) := \{x \in \mathbb{R} : x < \bar{x}_1(s)\}$ and $\mathcal{R}_2(s) := \{x \in \mathbb{R} : x > \bar{x}_2(s)\}$ denote the rejection sets for countries 1 and 2 and where X_i denotes the most generous credible settlement for a country i. The receiving country then accepts offers if and only if they are at least as good as their outside option. In particular, $y_i(x; s) = \mathbb{1}\{U_i(x; s) \geq W_i(s)\}$.

Note that the bargaining surplus is given by

$$B(s) = \begin{cases} \bar{x}_2(s) - \bar{x}_1(s) & \text{if } X_2 \geq \bar{x}_2(s) \geq \bar{x}_1(s) \geq X_1 \\ \bar{x}_2(s) - X_1 & \text{if } X_2 \geq \bar{x}_2(s) \geq X_1 > \bar{x}_1(s) \\ X_2 - \bar{x}_1(s) & \text{if } \bar{x}_2(s) > X_2 \geq \bar{x}_1(s) \geq X_1 \\ X_2 - X_1 & \text{if } \bar{x}_2(s) > X_2 \geq X_1 > \bar{x}_1(s) \\ 0 & \text{if otherwise.} \end{cases}$$

Therefore, note that equation (5) in the main text specifically refers to the primary case of interest where budget constraints do not interfere in any way. Since this is a necessary condition and budget constraints are fixed, I often refer to this object as the bargaining surplus more generally, and consider violations of the budget constraint ex post.

A.5 Competing for Bargaining Surplus

Competing is costly but improves the odds of recovering the bargaining surplus, which must be of common value to both countries. If there exists a bargaining surplus, $B(s) > 0$, both countries will cooperate and choose an amount of effort as a function of the state s. Then, a country i that's recognized as proposer will receive the value of the bargaining surplus today and possibly in the future, with likelihood according to λ. Hence, the expected net gain of recovering agenda-setting power becomes

$$B(s) + \delta\lambda B(s) + (\delta\lambda)^2 B(s) + \ldots = \frac{B(s)}{1-\delta\lambda}.$$

There is no equilibrium in pure effort strategies, as (1) countries have an incentive to increase their effort insofar as they are losing the competition and there exists profitable deviations above their opponent's level, (2) countries have incentive to decrease their effort insofar as they are losing and there is no profitable deviation above their opponent's level, and (3) countries that are winning have incentive to marginally decrease their effort.

Instead, consider equilibrium effort such that each country mixes according to a cumulative distribution function (c.d.f.) denoted by F_s^* in state s. Moreover, because the baseline model is a special case of the extended model with frictions in which $\mu = 0$, I focus on the general case here. In particular, I begin by looking for equilibrium diplomatic strategies where countries only exert nonzero efforts above any frictions that exist $\mu \geq 0$. By exerting effort $e \geq \mu$, country i then has an expected utility

$$\underbrace{W_i(s)}_{\substack{\text{expected}\\ \text{value of war}}} + \frac{B(s)}{1-\delta\lambda}\Big[\underbrace{F_s^*(e)\pi}_{\substack{\text{opponent}\\ \text{exerts less}}} + \underbrace{(1-F_s^*(e))(1-\pi)}_{\text{opponent exerts more}}\Big] - e. \tag{A10}$$

Each country's war payoff serves as their baseline value for cooperation, reflecting their outside option. By choosing to exert e on competition, a country can expect a share of the bargaining surplus according to the bracketed component, and have total value for it according to their discount on time δ and how long they expect the settlement to persist λ.

On the other hand, exerting zero effort yields an expected value

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\Big[\underbrace{F_s^*(\mu)\frac{1}{2}(1-\rho)}_{\text{mutual shirking}} + \underbrace{(1-F_s^*(\mu))(1-\pi)}_{\text{opponent exerts effort}}\Big] \tag{A11}$$

recalling from the main text that delay costs each country ρ of the gains from peace. The gain from peace to a country that exerts nothing reflects the odds they are still awarded the agenda-setting power in the event their opponent exerts effort. Mixing strategies require indifference between equations (A10) and (A11), allowing us to solve for the distribution F_s^* (refer to the proof of Proposition A1).

A.6 Equilibrium Characterization

This section formally states results that support and extend the analysis of the main text.

The below proposition provides a full characterization of equilibrium with strategies σ^* under primary focus in the main text.

Proposition A1 (Equilibrium). *An equilibrium exists where, for all $s \in S$ and $q : S \times A \to S$, each country $i = 1, 2$ plays $\sigma_i^*(s) = (a_i^*(s), e_i^*(s), x_i^*(s), y_i^*(x; s))$ defined as follows.*

(i) Initiate war $a_i^*(s) = 1$ if and only if $W_i(s) > U_i(s)$.

(ii) If $U_i(s) \geq W_i(s)$, choose effort $e_i^*(s)$ according to a mixed strategy with distribution

$$F_s^*(e) = \begin{cases} \frac{2\mu(1-\delta\lambda)}{(2\pi-(1-\rho))B(s)} & \text{for } e \in [0, \mu) \\ \frac{1-\delta\lambda}{(2\pi-1)B(s)}\left(e + \frac{2\pi-1-\rho}{2\pi-(1-\rho)}\mu\right) & \text{for } e \in \left[\mu, \frac{(2\pi-1)B(s)}{1-\delta\lambda} - \frac{2\pi-1-\rho}{2\pi-(1-\rho)}\mu\right] \end{cases}$$

with $F_s^*(e) = 0$ for $e < 0$ and $F_s^*(e) = 1$ for $e > \frac{(2\pi-1)B(s)}{1-\delta\lambda} - \frac{2\pi-1-\rho}{2\pi-(1-\rho)}\mu$. Otherwise exert zero effort $e_i^*(s) = 0$.

(iii) Offer $x_i^*(s)$ as given by equations (A8)-(A9) when recognized as agenda setter.

(iv) Accept an offer x, $y_i^*(x; s) = 1$, if and only if $U_i(x; s) \geq W_i(s)$.

Proof of Proposition A1. First, consider the war decision. The countries maximize their expected utility and if, given state s, their war continuation value $W_i(s)$ is larger than their continuation value from cooperating $U_i(s)$, they will necessarily prefer to fight. However, if there is positive bargaining surplus $B(s) \geq 0$, the expected net gain from exerting effort $e \geq 0$ on diplomacy is

$$\begin{aligned} \frac{B(s)}{1-\delta\lambda}\Big[&\pi \Pr(e > \max\{e_{-i}, \mu - e_{-i}\}) + (1-\pi) \\ &\times \Pr(e_{-i} > \max\{e, \mu - e\}) + \frac{1}{2}(1-\rho)\Pr(\mu > e + e_{-i})\Big] - e. \end{aligned} \tag{A12}$$

The net gain is zero when $B(s) = 0$, in which case neither country will be willing to exert a positive amount of effort in equilibrium. Therefore, to understand strategies with nonzero amounts of effort, assume $B(s) > 0$.

To see why there are no pure strategies in equilibrium, consider the following. If a country always exerts amount $e^* > 0$, their opponent would either deviate to an amount greater than e^* or zero. If the opponent deviated to zero, the country will prefer to exert less than e^*. If the opponent deviated to an amount greater than e^*, the country will prefer to move to a greater amount or deviate to zero. If both countries exert the same amount, they will either have an incentive to increase their effort a marginal amount to increase their gain by approximately double or they will prefer to deviate to zero.

I now look for a mixed strategy corresponding to state s given by c.d.f. F_s^* that satisfies equation (A12) for both countries. We can write the payoff from zero effort as

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[F_s^*(\mu)\frac{1}{2}(1-\rho) + (1-F_s^*(\mu))(1-\pi)\right]. \tag{A13}$$

On the other hand, exerting effort of μ yields an expected payoff

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[F_s^*(\mu)\pi + (1-F_s^*(\mu))(1-\pi)\right] - \mu. \tag{A14}$$

Using equations (A13) and (A14), we can solve for

$$F_s^*(\mu) = \frac{2\mu(1-\delta\lambda)}{(2\pi-(1-\rho))B(s)}. \tag{A15}$$

We know that μ cannot be the top of the support by assumption that frictions are not so high as to discourage cooperation altogether. Then, a country i exerting effort $e > \mu$ will receive an expected payoff

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[F_s^*(e)\pi + (1-F_s^*(e))(1-\pi)\right] - e. \tag{A16}$$

Using the indifference condition for equations (A14) and (A16) and plugging in equation (A15), we find that for $e \geq \mu$,

$$F_s^*(e) = \frac{1-\delta\lambda}{(2\pi-1)B(s)}\left(e + \frac{2\pi-1-\rho}{2\pi-(1-\rho)}\mu\right). \tag{A17}$$

By definition of a c.d.f., we know the largest amount a country can exert and still be indifferent is given by $\bar{e}_s := \inf\{e \geq 0 : F^*(\bar{e}_s) = 1\}$. Using equation (A17), we find that

$$\bar{e}_s = \frac{(2\pi-1)B(s)}{1-\delta\lambda} - \frac{2\pi-1-\rho}{2\pi-(1-\rho)}\mu.$$

Together, these equations yield an equilibrium effort strategy presented in Proposition A1. To check for profitable deviations, consider the case where a country exerts effort $e > \bar{e}_s$ with nonzero probability. By deviating, their payoff will be

$$W_i(s) + \frac{\pi B(s)}{1-\delta\lambda} - e < W_i(s) + \frac{\pi B(s)}{1-\delta\lambda} - \bar{e}_s,$$

and therefore they do not deviate. In words, a country already wins with certainty when expending $\bar{e}_s$, so there is no reason to ever exert more given their opponent plays this strategy as well.

Further, consider a deviation to expending a nonzero amount less than μ, $e \in (0, \mu)$, with some probability. By deviating, their expected payoff will be

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[F^*(\mu - e)\frac{1}{2}(1-\rho) + (1 - F^*(\mu - e))(1-\pi)\right] - e. \tag{A18}$$

Since their opponent is playing a strategy such that $F_s^*(\mu) = F^*(\mu - e)$ for any $e \in (0, \mu)$, we can plug this into equation (A18) and see that their payoff is equal to

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[F_s^*(\mu)\frac{1}{2}(1-\rho) + (1 - F_s^*(\mu))(1-\pi)\right] - e.$$

which is strictly less than their payoff from exerting zero given by equation (A13), hence this is not a profitable deviation. This is sufficient to show that F_s^* is an equilibrium effort strategy.

Equations (A6) and (A7) that govern equilibrium offers yield implicit conditions for $\bar{x}_2(s)$ and $\bar{x}_1(s)$, which represent the settlements at which country 1 and 2 are left indifferent between peace and war, respectively. Each country i will extend an offer equal to country $-i$'s indifference point or the greatest possible offer $-i$ can accept if and only if it is preferable to their own indifference point $\bar{x}_i(s)$ and the offer falls within their budgets $\mathcal{X}_i$. Otherwise, each country i will extend an offer that they know will get rejected, $x \in \mathcal{R}_{-i}(s)$ where $\mathcal{R}_i(s) := \{x : U_i(x; p) < W_i(s)\}$ denotes country i's rejection set in state s. A country i receiving an offer rejects offers in their rejection set and accepts all others. □

Additionally, this is the unique equilibrium when there are no frictions.

Proposition A2. *If $\mu = 0$, Proposition A1 characterizes the unique equilibrium.*

Proof of Proposition A2. Since the war, offer, and accept decisions are straightforward, we only need to show that there is no equilibrium under an alternative effort strategy $F' \neq F^*$.

First note that any effort strategy must be continuous. Suppose for example there is an F' with finite probability $\xi > 0$ of exerting effort e' in the distribution. Then, there exists a profitable deviation F'' in which a country could improve their expected share of the bargaining surplus by allocating greater probability to an arbitrarily small increase above e'. Thus, we know all effort strategies must be given by continuous distributions.

Second, zero must be in the support of the distribution. Suppose otherwise, so that the lower bound of the distribution is some $\underline{e} > 0$. Then, following this distribution would imply that countries exert strictly positive amounts despite an expectation that they will have the weakest performance. As a result, there would be a profitable deviation to effort less than $\underline{e}$ that save on the expense of competition without resulting in lower expected shares of the bargaining surplus.

Moreover, the top of the support must be equal to $\bar{e}_s$ as defined above. Suppose there is an alternate distribution F' such that $F'(\bar{e}_s) < 1$, i.e., countries exert greater than $\bar{e}_s$ with positive probability in equilibrium. Because we have just shown that zero must be in the support of F', expending amounts consistent with the new upper bound $\bar{\hat{e}} > \bar{e}_s = \frac{(2\pi-1)B(s)}{1-\delta\lambda}$ needs to be consistent with the lower bound of zero. Let $\bar{\hat{e}} = \bar{e}_s + \varepsilon$ for small $\varepsilon > 0$. This implies

$$W_i(s) + \frac{(1-\pi)B(s)}{1-\delta\lambda} = W_i(s) + \frac{\pi B(s)}{1-\delta\lambda} - \frac{(2\pi-1)B(s)}{1-\delta\lambda} - \varepsilon$$

which implies $\varepsilon = 0$, a contradiction. On the other hand, if there exists an $\bar{\hat{e}} < \bar{e}_s$ such that $F'(\bar{\hat{e}}) = 1$, there is a profitable deviation to exerting $\bar{\hat{e}} + \varepsilon$ for an arbitrarily small $\varepsilon > 0$.

Lastly, to show that the distribution is necessarily uniform, suppose for contradiction there is an alternate distribution F' where, for some $\hat{e} \in [0, \bar{e}_s]$, $F'(\hat{e}) > F^*(\hat{e}) = \frac{(1-\delta\lambda)\hat{e}}{(2\pi-1)B(s)}$. Then, the payoff from exerting $\hat{e}$ must be

$$\frac{B(s)}{1-\delta\lambda}\left[F'(\hat{e})\pi + (1-F'(\hat{e}))(1-\pi)\right] - \hat{e} = \frac{B(s)}{1-\delta\lambda}\left[F'(\hat{e})(2\pi-1) + (1-\pi)\right] - \hat{e} > \frac{(1-\pi)B(s)}{1-\delta\lambda}$$

which is the payoff from zero effort. But we know this must be in the support, a contradiction. The analogous argument can be made for F' such that $F'(\hat{e}) < F^*(\hat{e})$ for some $\hat{e} \in [0, \bar{e}_s]$. Hence, $\mu = 0$ implies that the equilibrium detailed in Proposition A1 is unique. □

A.7 Ex-Ante Continuation Value in an Absorbing State

This section shows that the ex-ante continuation value for an absorbing state can be expressed solely in terms of model primitives. This derivation is for purposes with $\mu = 0$, but a similar procedure can be followed to derive an analogous expression for when $\mu > 0$.

Take any state $s' \in S$ to be absorbing, i.e., $q(s'|s', a) = 1$ for any $a \in A$. Then, the aggregate present value associated with arriving in state s' is given by

$$V(s') = \frac{1}{1-\delta\lambda} - 2\int edF^*_{s'}(e) + \frac{\delta(1-\lambda)}{1-\delta\lambda}\left(\frac{1}{1-\delta\lambda} - 2\int edF^*_{s'}(e) + \dots\right).$$

Recognizing the geometric series, we can rewrite the expression as

$$V(s') = \frac{1 - 2(1-\delta\lambda)\int edF^*_{s'}(e)}{1-\delta}. \tag{A19}$$

Because s' is absorbing, we have a closed-form expression for its bargaining surplus. In particular, the bargaining surplus in an absorbing state is equal to

$$B(s') = 1 - (1-\delta\lambda)\left[\frac{1}{1-\delta\lambda} - C(s') + \frac{\delta(1-\lambda)}{1-\delta\lambda}V(s')\right] + \delta(1-\lambda)V(s') = (1-\delta\lambda)C(s')$$

where recall that $C(s) := c_1(s) + c_2(s)$. Consequently, we can now express a country's expected effort in an absorbing state s' in terms of the total costs of war in that state and model parameters,

$$\int edF^*_{s'}(e) = \frac{1}{2}(2\pi - 1)C(s').$$

Plugging this into equation (A19), we have

$$V(s') = \frac{1 - (1-\delta\lambda)(2\pi-1)C(s')}{1-\delta} \tag{A20}$$

yielding the continuation value for being in s' solely in terms of model primitives.

A.8 Costly Delay and Bargaining Surplus

There is a strong connection between the probability of costly delay and the surplus from cooperation: to understand the effect of a parameter on the former often requires an understanding of its effect on the latter. For example, the cost of delay in the event of coordination failure has a direct exogenous effect on the probability of costly delay, but also an indirect endogenous effect through the bargaining surplus and corresponding equilibrium behavior. The same is true for the persistence parameters. As a result, it is not straightforward to make claims about how changes to these parameters affect the probability of costly delay.

Whether one force or the other will prevail inevitably depends on the expected downstream consequences. For example, the total probability of costly delay is decreasing in the cost of delay if the changes to equilibrium behavior result in a large enough increase to the cooperative surplus.

Moreover, we can also see this inverse relationship borne out in the effect of peace deal reliability and war outcome durability.

Proposition A3. *Let $\lambda = \alpha\theta$. Then, (i) $\frac{\partial}{\partial\theta}\omega(s) \leq 0$ if and only if $\frac{\partial}{\partial\theta}B(s) \geq 0$,*

(ii) $\frac{\partial}{\partial\alpha}\omega(s) \leq 0$ if and only if $\frac{\partial}{\partial\alpha}B(s) \geq \frac{-\delta\theta B(s)}{1-\delta\lambda}$, and

(iii) $\frac{\partial}{\partial\rho}\omega(s) \leq 0$ if and only if $\frac{\partial}{\partial\rho}B(s) \geq \frac{B(s)}{2\pi-(1-\rho)}$.

Proof of Proposition A3. **(i) $\boldsymbol{\theta}$:** Taking the partial derivative of $\omega(s)$ with respect to θ,

$$\frac{\partial}{\partial\theta}\omega(s) = \frac{-8\mu^2(1-\delta\lambda)^2}{(2\pi-(1-\rho))^2 B(s)^3} \cdot \frac{\partial}{\partial\theta}B(s)$$

and because the first term must be negative, we can conclude that $\frac{\partial}{\partial\theta}\omega(s) \leq 0$ if and only if $\frac{\partial}{\partial\theta}B(s) \geq 0$. From the definition of $B(s)$, we can conclude $\frac{\partial}{\partial\theta}\omega(s) \leq 0$ if and only if

$$\frac{\partial}{\partial\theta}B(s) = \delta(1-\lambda)\sum_{s'}\frac{\partial}{\partial\theta}V(s')q(s'|s,a_U) - (1-\delta\lambda)\frac{\partial}{\partial\theta}W(s) \geq 0$$

or equivalently $\delta(1-\lambda)\sum_{s'}\frac{\partial}{\partial\theta}V(s')q(s'|s,a_U) > (1-\delta\lambda)\frac{\partial}{\partial\theta}W(s)$.

(ii) $\boldsymbol{\alpha}$: Plugging in $\alpha\theta$ for λ and taking the partial derivative of $\omega(s)$,

$$\frac{\partial}{\partial\alpha}\omega(s) = \frac{-8\mu^2(1-\delta\alpha\theta)(\delta\theta B(s) + (1-\delta\alpha\theta)\frac{\partial}{\partial\alpha}B(s))}{(2\pi-(1-\rho))^2 B(s)^2}.$$

This expression is negative if and only if $\frac{\partial}{\partial\alpha}B(s) \geq \frac{-\delta\theta B(s)}{1-\delta\theta}$ for any valid parameter values.

(iii) $\boldsymbol{\rho}$: Taking the partial derivative of $\omega(s)$ with respect to ρ,

$$\frac{\partial}{\partial\rho}\omega(s) = \frac{-8\mu^2(1-\delta\lambda)^2(B(s) + (2\pi-(1-\rho))\frac{\partial}{\partial\rho}B(s))}{(2\pi-(1-\rho))^3 B(s)^3)}.$$

This expression is negative if and only if $\frac{\partial}{\partial\rho}B(s) \geq \frac{B(s)}{2\pi-(1-\rho)}$. Refer to the supplementary files for verification of these derivatives and inequalities. □

To see how these results depend on downstream expectations, it is useful to observe that the effect of a parameter on the bargaining surplus is inextricably tied to its differential effect on a country's incentive to fight. In the case of θ, we can see that $\frac{\partial}{\partial\theta}B(s) \geq 0$ if and only if $\delta(1-\lambda)\sum_{s'}\frac{\partial}{\partial\theta}V(s')q(s'|s,a_U) \geq (1-\delta\lambda)\frac{\partial}{\partial\theta}W(s)$ for any state $s \in S$, by definition. On the left-hand side of this equation reflects the effect of war outcome durability on the expected continuation values upon a return to bargaining after cooperation in state s, whereas the right-hand side reflects its effect on the continuation values for war in state s. The probability of costly delay is inversely related to the surplus, and the surplus is increasing if future values after cooperating are expanding at a fast enough rate relative to that of the current war value. Clearly, the way in which actions in the current state of the world map onto probability distributions over future states of the world is paramount in determining whether the inequality will hold.

B Proofs

Proof of Lemma 1. Without loss of generality, suppose country 1 is the agenda setter. Then, recall from Appendix A.4 that the equilibrium offer is equal to

$$x_1^*(s) = 1 - (1-\delta\lambda)W_2(s) + \delta(1-\lambda)\sum_{s'\in S} V_2(s')q(s'|s,a_U)$$

if $X_2 \geq \bar{x}_2(s) \geq \bar{x}_1(s)$, or else $x_1^*(s) = X_2$ if $\bar{x}_2(s) > X_2 \geq \bar{x}_1(s)$ or $x_1^*(s) = x$ for any $x \in \mathcal{R}_2(s)$ otherwise. In the latter two cases, there is no direct effect of θ or λ. Therefore, suppose $X_2 \geq \bar{x}_2(s) \geq \bar{x}_1(s)$, which reflects the primary setting of interest: on-path equilibrium behavior that is unaffected by budget constraints. Unpacking the continuation values, we can see that

$$x_1^*(s) = \frac{\delta(\lambda-\theta)}{1-\delta\theta} - (1-\delta\lambda)c_2(s) + \sum_{s'} V_2(s')\left[\frac{\delta(1-\theta)(1-\delta\lambda)}{1-\delta\theta}q(s'|s,a_W) + \delta(1-\lambda)q(s'|s,a_U)\right]$$

Taking the partial derivative with respect to θ:

$$\frac{\partial}{\partial\theta}x_1^*(s) = \underbrace{\frac{-\delta(1-\delta\lambda)}{(1-\delta\theta)^2}}_{\text{direct}} + \underbrace{\frac{\partial}{\partial\theta}\left(\sum_{s'} V_2(s')\left[\frac{\delta(1-\theta)(1-\delta\lambda)}{1-\delta\theta}q(s'|s,a_W) + \delta(1-\lambda)q(s'|s,a_U)\right]\right)}_{\text{indirect}}$$

where we can see the direct effect must be negative under any configuration of parameters.

Next, taking the partial derivative with respect to λ:

$$\frac{\partial}{\partial\lambda}x_1^*(s) = \underbrace{\frac{1}{1-\delta\theta} + \delta c_2(s)}_{\text{direct}} + \underbrace{\frac{\partial}{\partial\lambda}\left(\sum_{s'} V_2(s')\left[\frac{\delta(1-\theta)(1-\delta\lambda)}{1-\delta\theta}q(s'|s,a_W) + \delta(1-\lambda)q(s'|s,a_U)\right]\right)}_{\text{indirect}}$$

from which the direct effect must be positive. Assuming the direct effects prevail ensures that country 1 can extract greater concessions from country 2 in equilibrium when θ is lower and λ is higher. By symmetry country 2 can extract greater concessions under these same circumstances, from which we can conclude agenda-setting power is more valuable. $\square$

Proof of Proposition 1. In the absence of competitive diplomacy, we can suppose, for example, that $\pi = \frac{1}{2}$ and therefore the agenda setter is randomly recognized with probability $\frac{1}{2}$ regardless of their diplomatic effort. Any other rule that does not reward competition, such as deterministic agenda setter endowments, are also acceptable. From there, it follows that countries will not exert costly effort for no gain, and hence $e_1(s) = e_2(s) = 0$ for all states s.

Since cooperation then does not entail inefficiencies, it must be that for any period s, the sum of continuation values under an always cooperate strategy σ_U is $V^{\sigma_U}(s) = V_1^{\sigma_U}(s) + V_2^{\sigma_U}(s) = \frac{1}{1-\delta}$ for any state $s \in S$. Further, since war is inherently costly, the sum of the continuation values in under any strategy σ_W that involves some war must be strictly less than this amount, $V^{\sigma_W}(s) < V^{\sigma_U}(s)$ for any state s. Let $\hat{s}$ be a state where at least one country prefers to fight a war in equilibrium strategy σ^*. Then, we know $V^{\sigma^*}(\hat{s}) < V^{\sigma_U}(\hat{s})$ which in turn implies $V_i^{\sigma^*}(\hat{s}) < V_i^{\sigma_U}(\hat{s})$ for at least one country i. Since war occurs in state $\hat{s}$, we know that for this country $W_i^{\sigma^*}(\hat{s}) > U_i^{\sigma^*}(\hat{s})$.

Now suppose country $-i$ is recognized as agenda setter in state $\hat{s}$. By extending equilibrium offer according to σ^*, it will be rejected and war will ensue, leaving them with a payoff $W_{-i}^{\sigma^*}(\hat{s})$. However, we know that, due to the inefficiency of war, countries must lose out on a total surplus of $\frac{1}{1-\delta} - V(\hat{s}) > 0$. Therefore, there is a strictly profitable deviation to strategy σ^{**} with offer $x^{**} \in \mathbb{R}$ such that $U_i(x^{**}; \hat{s}) = W_i(\hat{s})$, allowing $-i$ to consume the additional surplus $U_{-i}^{\sigma^{**}}(\hat{s}) = W_{-i}^{\sigma^*}(\hat{s}) + \frac{1}{1-\delta} - V^{\sigma^*}(\hat{s})$. Country $-i$ would be strictly better off by playing this strategy and country i would accept this offer as it satisfies their indifference condition. Further, since country $-i$ is willing to extend this offer, country i will be able to recover a settlement at least as good as x^{**} in the event they are recognized as agenda setter. $\square$

Proof of Proposition 2. By Proposition A1, $\mu = 0$ implies

$$\frac{1-\delta\lambda}{(2\pi-1)B(s)}\left(e + \frac{2\pi-1-\rho}{2\pi-(1-\rho)}\mu\right) = \frac{(1-\delta\lambda)e}{(2\pi-1)B(s)}$$

for all $e \in [0, \bar{e}_s^{\mu=0}]$ and some $\bar{e}_s^{\mu=0} \geq 0$. The upper bound can be solved by observing that $F_s^*(\bar{e}_s^{\mu=0}) = 1$, which yields

$$\bar{e}_s^{\mu=0} = \frac{(2\pi-1)B(s)}{1-\delta\lambda}.$$

By Proposition A2, this equilibrium is unique. □

Proof of Proposition 3. First, define the following set of terms for notational convenience. Let $C(s) := c_1(s) + c_2(s)$ reflect the total costs of war in state s, $W(s) = \sum_i W_i(s)$ the sum of the war continuation values in state s, $\hat{W}(s) := W(s) + C(s)$ the sum of the war continuation values less the total costs of war in state s, $U(s) := \sum_i U_i(s)$ the sum of the cooperation continuation values in state s, and $V(s) := \sum_i V_i(s)$ the sum of the ex-ante continuation values in state s.

Part 1. Costs of war: Recall that total expected effort $\int edF_s^*(e) = \frac{(2\pi-1)B(s)}{2(1-\delta\lambda)}$, where

$$B(s) = 1 - (1-\delta\lambda)W(s) + \delta(1-\lambda)\sum_{s'\in S} V(s')q(s'|s, a_U)$$

defines the bargaining surplus for all $s, s' \in S$, for both countries $i = 1, 2$. Then, plugging in for $B(s)$, total effort allocated to competitive diplomacy in state s is

$$E(s) := 2\int edF_s^*(e) = \frac{2\pi-1}{1-\delta\lambda}\left[1 - (1-\delta\lambda)W(s) + \delta(1-\lambda)\sum_{s'\in S} V(s')q(s'|s, a_U)\right].$$

Note that $\frac{\partial}{\partial C(s)}E(s) = 0$ for any $s \in S$ such that $V(s) = W(s)$ and hence $E(s) = 0$. Letting $s_{min} := \arg\min_s\{\frac{\partial}{\partial C(s)}E(s) : V(s) = U(s)\}$ and $C(s_{min}) := C$, we can take the partial derivative,

$$\begin{aligned}
\frac{\partial}{\partial C}E(s_{min}) &= \frac{2\pi-1}{1-\delta\lambda}\left[(1-\delta\lambda)\left(1 - \frac{\partial}{\partial C}\hat{W}(s_{min})\right) + \delta(1-\lambda)\sum_{s'\in S}\frac{\partial}{\partial C}U(s')q(s'|s_{min}, a_U)\right] \\
&= \frac{2\pi-1}{1-\delta\lambda}\left[(1-\delta\lambda)\left(1 - \frac{\delta(1-\theta)}{1-\delta\theta}\frac{\partial}{\partial C}U(s_{min})\right) + \delta(1-\lambda)\frac{\partial}{\partial C}U(s_{min})\right] \\
&= 2\pi - 1 + \left[\frac{(2\pi-1)\left(\delta(1-\lambda) - \frac{\delta(1-\delta\lambda)(1-\theta)}{1-\delta\theta}\right)}{1-\delta\lambda}\right]\frac{\partial}{\partial C}U(s_{min})
\end{aligned}$$

where we assume that $q(s_{min}|p, a) = 1$ for all $s \in S$ to impose the worst case scenario if it is possible for $\frac{\partial}{\partial C(s)}E(s) < 0$. If the partial derivative of $E(s_{min})$ with respect to C is positive under these conditions, then it must be positive in all cases.

Plugging in for $\frac{\partial}{\partial C}U(s_{min})$, we have

$$\begin{aligned}\frac{\partial}{\partial C}E(s_{min}) &= 2\pi - 1 + \frac{(2\pi-1)\left(\delta(1-\lambda) - \frac{\delta(1-\delta\lambda)(1-\theta)}{1-\delta\theta}\right)}{1-\delta\lambda}\left(-\frac{\partial}{\partial C}E(s_{min}) + \frac{\delta(1-\lambda)}{1-\delta\lambda}\frac{\partial}{\partial C}U(s_{min})\right)\\ &= 2\pi - 1 + \frac{(2\pi-1)\left(\delta(1-\lambda) - \frac{\delta(1-\delta\lambda)(1-\theta)}{1-\delta\theta}\right)}{1-\delta\lambda}\\ &\qquad \times\left(-\frac{\partial}{\partial C}E(s_{min}) + \frac{\delta(1-\lambda)}{1-\delta\lambda}\left(-\frac{\partial}{\partial C}E(s_{min}) + \frac{\delta(1-\lambda)}{1-\delta\lambda}\Big(\dots\Big)\right)\right).\end{aligned}$$

Noting the geometric series, we can express the above as

$$\frac{\partial}{\partial C}E(s_{min}) = 2\pi - 1 + \frac{(2\pi-1)\left(\delta(1-\lambda) - \frac{\delta(1-\delta\lambda)(1-\theta)}{1-\delta\theta}\right)}{1-\delta\lambda}\left(\frac{-(1-\delta\lambda)\frac{\partial}{\partial C}E(s_{min})}{1-\delta}\right) \tag{A21}$$

Solving for $\frac{\partial}{\partial C}E(s_{min})$,

$$\frac{\partial}{\partial C}E(s_{min}) = \left[1 - \frac{\delta(1-\delta)^2(2\pi-1)(\lambda-\theta)}{(1-\delta\lambda)^2(1-\delta\theta)}\right]^{-1}(2\pi-1) > 0$$

which is strictly positive for all valid parameter values.

Part 2. Welfare: Define $s_{i,max} := \arg\max_s \frac{\partial}{\partial c_i(s)}V_i(s)$. If $V_i(s_{i,max}) = W_i(s_{i,max})$, it is straightforward to see that

$$\begin{aligned}\frac{\partial}{\partial c_i(s_{i,max})}W_i(s_{i,max}) &= -1 + \frac{\delta(1-\theta)}{1-\delta\theta}\sum_{s'\in S}\frac{\partial}{\partial c_i(s_{i,max})}V(s')q(s'|s_{i,max},a_W)\\ &\leq -1 + \frac{\delta(1-\theta)}{1-\delta\theta}\frac{\partial}{\partial c_i(s_{i,max})}W_i(s_{i,max})\end{aligned}$$

implying

$$\frac{\partial}{\partial c_i(s_{i,max})}W_i(s_{i,max}) \leq -\frac{1-\delta\theta}{1-\delta} < 0.$$

On the other hand, if $V_i(s_{i,max}) = U_i(s_{i,max})$, we have

$$
\begin{aligned}
\frac{\partial}{\partial c_i(s_{i,max})} U_i(s_{i,max}) &= -\frac{\partial}{\partial c_i(s)} \int e dF_s^*(e) + \frac{\delta(1-\lambda)}{1-\delta\lambda} \sum_{s' \in S} \frac{\partial}{\partial c_i(s_{i,max})} V(s') q(s'|s_{i,max}, a_W) \\
&\leq -\frac{\partial}{\partial c_i(s)} \int e dF_s^*(e) + \frac{\delta(1-\lambda)}{1-\delta\lambda} \frac{\partial}{\partial c_i(s_{i,max})} U_i(s_{i,max}) \\
&\leq \frac{\delta(1-\lambda)}{1-\delta\lambda} \frac{\partial}{\partial c_i(s_{i,max})} U_i(s_{i,max}),
\end{aligned}
$$

where the final inequalty follows from the first part of this proof. This condition requires

$$
\frac{\partial}{\partial c_i(s_{i,max})} U_i(s_{i,max}) = 0.
$$

□

Proof of Proposition 4.

Part 1. Partial derivative: *Step 1. Show that* $\frac{\partial}{\partial \pi} B(s) \leq 0$ *and* $\frac{\partial}{\partial \pi} \int e dF_s^*(e) \geq 0$ *for all* $s \in S$. Defining $E(s) := 2 \int e dF_s^*(e)$, we can expand

$$
E(s) = \left(\frac{(2\pi - 1) B(s)}{1 - \delta\lambda} - \frac{2\pi - 1 - \rho}{2\pi - (1-\rho)} \mu + \mu \right) \times \left(1 - \frac{2\mu(1-\delta\lambda)}{(2\pi - (1-\rho)) B(s)} \right).
$$

This implies the partial derivative with respect π is

$$
\begin{aligned}
\frac{\partial}{\partial \pi} E(s) = &\left(1 - \frac{2\mu(1-\delta\lambda)}{(2\pi - (1-\rho)) B(s)} \right) \left(\frac{2B(s)}{1-\delta\lambda} + \frac{(2\pi - 1)\frac{\partial}{\partial \pi} B(s)}{1-\delta\lambda} - \frac{4\mu\rho}{2\pi - (1-\rho))^2} \right) \\
&- \left(2\mu(2\mu\rho(\delta\lambda - 1) - (2\pi - 1)(2\pi - (1-\rho)) B(s))(2B(s) + (2\pi - (1-\rho)) B(s)) \right) \\
&\bigg/ \left((2\pi - (1-\rho))^3 B(s)^2 \right) \qquad \text{(A22)}
\end{aligned}
$$

Equation (A22) implies that $\frac{\partial}{\partial \pi} B(s) \geq 0$ is a sufficient condition[30] to show that $\frac{\partial}{\partial \pi} E(s) \geq 0$ for all $s \in S$ and for any transition function $q : S \times A \to S$.

Suppose there exists s and q such that $\frac{\partial}{\partial \pi} E(s) < 0$. By definition of $B(s)$, we can write

$$
\frac{\partial}{\partial \pi} B(s) = \delta(1-\lambda) \sum_{s'} \frac{\partial}{\partial \pi} V(s') q(s'|s, a_U) - (1-\delta\lambda) \frac{\partial}{\partial \pi} W(s).
$$

[30] See the supplementary files for the precise threshold that gives a necessary and sufficient condition in terms of parameters, which must be strictly negative.

The expression is decreasing in the partial derivative of $W(s)$ and increasing in the partial derivative of the expected ex-ante continuation value tomorrow from state s. Note that

$$\frac{\partial}{\partial\pi}W(s) = \frac{\delta(1-\theta)}{1-\delta\theta}\sum_{s'}\frac{\partial}{\partial\pi}V(s')q(s'|s,a_W)$$
$$\frac{\partial}{\partial\pi}U(s) = -\frac{\partial}{\partial\pi}E(s) + \frac{\delta(1-\lambda)}{1-\delta\lambda}\sum_{s'}\frac{\partial}{\partial\pi}V(s')q(s'|s,a_U).$$

Let $s_{min} := \arg\min_s \frac{\partial}{\partial\pi}B(s)$. Hence, because $-\frac{\partial}{\partial\pi}E(s) > 0$, the minimum value of $\frac{\partial}{\partial\pi}B(s_{min})$ cannot exceed the case where $V(s_{min}) = W(s_{min})$ and $q(s_{min}|s_{min}, a_W) = 1$, or equivalently $\frac{\partial}{\partial\pi}B(s_{min}) = 0$.

Therefore, since $\frac{\partial}{\partial\pi}E(s) < 0$ implies $\frac{\partial}{\partial\pi}B(s) \geq 0$, which in turn implies $\frac{\partial}{\partial\pi}E(s) \geq 0$, we can conclude $\frac{\partial}{\partial\pi}E(s) \geq 0$. By the same logic as above, replacing s_{min} for $s_{max} := \arg\max_s \frac{\partial}{\partial\pi}B(s)$ and solving, we have $\frac{\partial}{\partial\pi}B(s) \leq 0$ for all $s \in S$.

Step 2. Show that $\frac{\partial}{\partial\pi}V_i(s) \leq 0$ for all $s \in S$. Let $s_{i,max} = \arg\max_s \frac{\partial}{\partial\pi}V_i(s)$. If $V_i(s_{i,max}) = W_i(s_{i,max})$ then

$$\begin{aligned}\frac{\partial}{\partial\pi}W_i(s_{i,max}) &= \frac{\delta(1-\theta)}{1-\delta\theta}\sum_{s'\in S}\frac{\partial}{\partial\pi}V_i(s')q(s'|s_{i,max},a_W)\\ &\leq \frac{\delta(1-\theta)}{1-\delta\theta}\frac{\partial}{\partial\pi}W_i(s_{i,max})\end{aligned}$$

which is only satisfied by $\frac{\partial}{\partial\pi}W_i(s_{i,max}) = 0$.

On the other hand, $V_i(s_{i,max}) = U_i(s_{i,max})$ implies

$$\begin{aligned}\frac{\partial}{\partial\pi}U_i(s_{i,max}) &= -\frac{\partial}{\partial\pi}E(s_{i,max}) + \frac{\delta(1-\lambda)}{1-\delta\lambda}\sum_{s'\in S}\frac{\partial}{\partial\pi}V_i(s')q(s'|s_{i,max},a_U)\\ &\leq -\frac{\partial}{\partial\pi}E(s_{i,max}) + \frac{\delta(1-\lambda)}{1-\delta\lambda}\frac{\partial}{\partial\pi}U_i(s_{i,max})\end{aligned}$$

which implies

$$\frac{\partial}{\partial\pi}U_i(s_{i,max}) \leq \frac{-(1-\delta\lambda)\frac{\partial}{\partial\pi}E(s_{i,max})}{1-\delta} \leq 0.$$

Part 2. Completely decisive contests: First note that if $U_i(s) < W_i(s)$ then the result is trivially satisfied. Suppose then that $U_i(s) \geq W_i(s)$.

Recall $U_i(s)$ can be expressed by the expected value of exerting zero effort

$$U_i(s) = W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[F_s^*(\mu)\frac{1}{2}(1-\rho) + (1-F_s^*(\mu))(1-\pi)\right]. \tag{A23}$$

Additionally, recall from Proposition A1 the definition of $F_s^*(\mu)$. Plugging into (A23) yields

$$U_i(s) = W_i(s) + \frac{B(s)}{1-\delta\lambda}\left[1-\pi + \frac{2\mu(1-\delta\lambda)}{(2\pi-(1-\rho))B(s)}\left(\frac{1}{2}(1-\rho)-(1-\pi)\right)\right]$$

and setting $\pi = 1$ yields $U_i(s) = W_i(s) + \frac{1-\rho}{1+\rho}\mu$, concluding the proof. $\square$

Proof of Proposition 5. By Definition 1, $\bar{x}_1(s) > \bar{x}_2(s)$ implies that war is efficient. By equations (A6) and (A7), this is equivalent to

$$\frac{1}{1-\delta\lambda} - \frac{1}{1-\delta\theta} + \sum_{s'\in S} V(s')\left[\frac{\delta(1-\lambda)}{1-\delta\lambda}q(s'|s,a_U) - \frac{\delta(1-\theta)}{1-\delta\theta}q(s'|s,a_W)\right] + C(s) < 0,$$

yielding the desired equation by definition of G and $\Delta V(s)$. $\square$

Proof of Corollary 1. Recall that efficient wars break out if and only if $\bar{x}_1(s) > \bar{x}_2(s)$, or equivalently

$$1 - (1-\delta\lambda)W(s) + \delta(1-\lambda)\sum_{s'} V(s')q(s'|s,a_U) < 0.$$

Plugging in for $W(s)$, this yields the following condition in terms of the total costs of war,

$$C(s) < \frac{1}{1-\delta\theta} - \frac{1}{1-\delta\lambda} + \frac{\delta(1-\theta)}{1-\delta\theta}\sum_{s'} V(s')q(s'|s,a_W) - \frac{\delta(1-\lambda)}{1-\delta\lambda}\sum_{s'} V(s')q(s'|s,a_U).$$

Letting $\lambda = 0$ and $\theta = 1$, the condition becomes

$$C(s) < \frac{\delta}{1-\delta} - \sum_{s'} V(s')q(s'|s,a_U).$$

Then, we can expand the expression to

$$C(s) < \frac{\delta}{1-\delta} - \sum_{s'}\left[1-\tau(s') + \delta\sum_{s''} V(s'')q(s''|p',a_U)\right]q(s'|s,a_U)$$

where $\tau(s) := 2\int edF_s^*(e)$ for all s such that $V(s) = U(s)$ and $\tau(s) := C(s)$ for all s such that $V(s) = W(s)$. Letting $\hat{\tau}_s := \sum_{s'} \tau(s')q(s'|s, a_U)$ denote the expected inefficiency upon returning to bargaining after cooperation in state s (across both diplomatic effort and costs of war depending on which arises), we can rewrite the condition as

$$C(s) < \frac{\delta}{1-\delta} + \hat{\tau}_s - 1 - \delta \sum_{s'} \sum_{s''} V(s'')q(s''|s', a_U)q(s'|s, a_U)$$

Choose any $\tau \in (0, \hat{\tau}_s)$. Then, we have a sufficient condition

$$\begin{aligned} C(s) &< \frac{\delta}{1-\delta} + \tau - 1 - \delta \sum_{s'} \sum_{s''} V(s'')q(s''|p', a_U)q(s'|s, a_U) \\ &< \frac{\delta}{1-\delta} + \tau - 1 - \delta \sum_{s'} \sum_{s''} \left[1 - \hat{\tau}_{s''} + \delta \sum_{s'''} V(s''')q(s'''|p', a_U)\right] q(s''|p', a_U)q(s'|s, a_U). \end{aligned}$$

Again choosing a $\tau \in (0, \hat{\tau}_{s''})$, we can write a sufficient condition as

$$C(s) < \frac{\delta}{1-\delta} + (\tau - 1)(1+\delta) - \delta^2 \sum_{s'} \sum_{s''} \sum_{s'''} V(s''')q(s'''|p', a_U)q(s''|p', a_U)q(s'|s, a_U).$$

Following this geometric series, war is directly preferred to cooperation if

$$C(s) < \frac{\delta + \tau - 1}{1-\delta} \tag{A24}$$

Because $\lim_{\delta \to 1^-} \frac{\delta+\tau-1}{1-\delta} = \tau\infty$ and $\tau > 0$, inequality (A24) holds for sufficiently large δ. $\square$

Proof of Corollary 2. Recall from the proof of Corollary 1 that a general condition for efficient war is

$$C(s) < \frac{1}{1-\delta\theta} - \frac{1}{1-\delta\lambda} + \frac{\delta(1-\theta)}{1-\delta\theta} \sum_{s'} V(s')q(s'|s, a_W) - \frac{\delta(1-\lambda)}{1-\delta\lambda} \sum_{s'} V(s')q(s'|s, a_U)$$

for any $s \in S$ and $q : S \times A \to S$.

If we know that q satisfies $\sum_{s'} V(s')q(s'|s, a_U) \geq \sum_{s'} V(s')q(s'|s, a_W)$ for all s, then we can deduce the following sufficient condition,

$$C(s) < \frac{1}{1-\delta\theta} - \frac{1}{1-\delta\lambda} + \left[\frac{\delta(1-\theta)}{1-\delta\theta} - \frac{\delta(1-\lambda)}{1-\delta\lambda}\right] \sum_{s'} V(s')q(s'|s, a_W).$$

We know that $C(s) > 0$ and $\sum_{s'} V(s')q(s'|s, a_W) \leq \frac{1}{1-\delta}$ for all $s \in S$. When $\lambda > \theta$, the first component (G) is strictly negative and the second bracketed component is strictly positive. Then, a necessary condition for efficient war becomes

$$C(s) < \frac{1}{1-\delta\theta} - \frac{1}{1-\delta\lambda} + \frac{1}{1-\delta}\left[\frac{\delta(1-\theta)}{1-\delta\theta} - \frac{\delta(1-\lambda)}{1-\delta\lambda}\right] = 0,$$

which cannot hold. □

Proof of Proposition 6. Refer to the proof of Proposition A1. □

Proof of Proposition 7. Refer to the proofs of Propositions 3, 4, and 5, which consider $\mu > 0$. □

Proof of Proposition 8. By the definition of $\omega(s)$, we have

$$\frac{\partial}{\partial\pi}\omega(s) = \frac{-8\mu^2(1-\delta\lambda)^2}{(2\pi-(1-\rho))^2 B(s)^2}\left[\frac{2}{2\pi-(1-\rho)} + \frac{\frac{\partial}{\partial\pi}B(s)}{B(s)}\right]$$

For this condition to be negative, we require

$$\frac{\partial}{\partial\pi}B(s) \geq \frac{-2B(s)}{2\pi-(1-\rho)} := D_*$$

where D_* denotes the floor of $\frac{\partial}{\partial\pi}B(s)$ if the probability of war is to decrease in competition. Note that, for given any valid set of parameters $(\delta, \lambda, \theta, \rho, \mu)$, any transition function q, and any consequent bargaining surplus $B(s)$, it must be that $D_* < 0$.

Next, recall that by the definition of $B(s)$, we have

$$\begin{aligned}
\frac{\partial}{\partial\pi}B(s) &= \delta(1-\lambda)\sum_{s'}\frac{\partial}{\partial\pi}V(s')q(s'|s,a_U) - (1-\delta\lambda)\frac{\partial}{\partial\pi}W(s) \\
&= \sum_{s'}\frac{\partial}{\partial\pi}V(s')\left[\delta(1-\lambda)q(s'|s,a_U) - \frac{\delta(1-\theta)(1-\delta\lambda)}{1-\delta\theta}q(s'|s,a_W)\right] \\
&\geq \frac{\partial}{\partial\pi}V(s_{min})\delta(1-\lambda) - \frac{\partial}{\partial\pi}V(s_{max})\frac{\delta(1-\theta)(1-\delta\lambda)}{1-\delta\theta}
\end{aligned}$$

where $s_{min} := \arg\min_s \frac{\partial}{\partial\pi}B(s)$ and $s_{max} := \arg\max_s \frac{\partial}{\partial\pi}B(s)$. Then, with $\lambda = 1$ this becomes a lower bound of $-\frac{\partial}{\partial\pi}V(s_{max})\frac{\delta(1-\theta)(1-\delta)}{1-\delta\theta}$. Recall by Proposition 4 that $\frac{\partial}{\partial\pi}V(s) \leq 0$ for all s and q and hence the lower bound on $\frac{\partial}{\partial\pi}B(s)$ is weakly positive when $\lambda = 1$. As a result, we know that $1 = \lambda \geq \theta$ implies $\frac{\partial}{\partial\pi}B(s) \geq 0$, which additionally implies that $\frac{\partial}{\partial\pi}B(s) \geq D_*$. □

C Robustness

C.1 Competitive Diplomacy as a Tullock Contest

In the main text, a country is recognized as proposer with probability $\pi > \frac{1}{2}$ when they exert greater effort in diplomacy. Work in economic theory has demonstrated that many contests are strategically equivalent (see Chowdhury and Sheremeta (2015), for example). In this section, I show that the core incentives that drive strategic behavior in the all-pay contest of the main text carries into any decisive Tullock contest.

In particular, consider instead that endogenous country efforts govern the likelihood of having the upper-hand in the bargaining game, so that country i can earn up to π probability of recovering proposal power according to its share of total diplomatic efforts. Then, country i's payoff to exerting effort $e_i \geq 0$ when their adversary exerts $e^*_{-i} \geq 0$ becomes

$$W_i(s) + \xi_i(e_i, e^*_{-i}; d)\frac{(2\pi - 1)B(s)}{1 - \delta\lambda} + \frac{(1 - \pi)B(s)}{1 - \delta\lambda} - e_i$$

where

$$\xi_i(e_i, e_{-i}; d) = \begin{cases} \frac{e_i^d}{e_i^d + e_{-i}^d} & \text{if } e_i + e_{-i} > 0 \\ \frac{1}{2} & \text{otherwise} \end{cases}$$

for some decisiveness parameter $d \geq 0$. Importantly, note that the decisiveness parameter d refers to the decisiveness of the Tullock contest, not to be confused with the decisiveness parameter π refers to the decisiveness of performance on recognition of agenda setting power. Tullock contests are defined by d, but our parameter of interest in the main text is π.

The next proposition immediately follows Theorem 4.1 of Ewerhart (2015), with the only change being that competition is now over the long-run expected value of the bargaining surplus instead of a unit. The proof is rederived and stated for completeness.

Proposition A4. *In any equilibrium with $2 < d < \infty$, the support of the distribution of effort has zero as an accumulation point. The equilibrium is characterized by a sequence $e_1 > e_2 > \ldots > 0$*

with $\lim_{k\to\infty} e_k = 0$ *chosen with respective probabilities* f_1, f_2, ... *such that* $\sum_k f_k = 1$. *Moreover,*

$$\frac{(2\pi-1)B(s)}{1-\delta\lambda}\sum_k \frac{f_k e_j^d}{e_j^d+e_k^d} - e_j = 0 \tag{A25}$$

$$\frac{(2\pi-1)B(s)}{1-\delta\lambda}\sum_k \frac{f_k d e_j^{d-1} e_k^d}{(e_j^d+e_k^d)^2} - 1 = 0 \tag{A26}$$

for any integer $j \geq 1$, $s \in S$.

Proof of Proposition A4. This proof is derivative of Ewerhart (2015) with the only change being that competition is over the long-run expected value of the bargaining surplus instead of a unit, and is restated here only for the sake of completeness.

Let $e_1 > e_2 > \ldots > e_L$ be the mass points of the equilibrium effort distribution, used with respective probabilities $f_1, \ldots, f_L$, $\sum_{k=1}^L f_k = 1$. Suppose for contradiction that zero is not an accumulation point of the support. From the first-order condition at e_L,

$$\frac{(2\pi-1)B(s)}{1-\delta\lambda}\sum_{k=1}^L \frac{f_k d e_L^{d-1} e_k^d}{(e_L^d+e_k^d)^2} = 1,$$

we recover

$$\frac{(2\pi-1)B(s)}{1-\delta\lambda}\sum_{k=1}^L \frac{2 f_k e_k^d e_L^d}{(e_L^d+e_k^d)^2} - e_L = \frac{(2-d)e_L}{d}.$$

But since $e_k \geq e_L$ for all $k = 1, \ldots, L$, it follows that there is an upper bound on the expected payoff of exerting effort e_L,

$$\frac{(2\pi-1)B(s)}{1-\delta\lambda}\sum_{k=1}^L \frac{f_k e_L^d}{e_L^d+e_k^d} - e_L \leq \frac{(2-d)e_L}{d}.$$

Therefore, $d > 2$ implies a negative expected payoff for any equilibrium level of effort $e_L > 0$, a contradiction.

To prove equation (A25), taking any index $j \geq 1$, letting $j \to \infty$, and subsequently exchanging the sum and the limit via Lebesgue's Dominated Convergence Theorem, we recover that the expected equilibrium payoff is zero,

$$\frac{(2\pi-1)B(s)}{1-\delta\lambda}\sum_{k=1}^\infty \frac{f_k e_j^d}{e_j^d+e_k^d} - e_j = 0,$$

completing the proof. $\square$

While equation (A26) defines the first-order conditions, equation (A25) demonstrates that all decisive ($d > 2$) Tullock contests erode the net gain from being recognized as the agenda setter, which is increasing in π and results in complete depletion of the entire bargaining surplus at $\pi = 1$, consistent with behavior in the main text. Straightforwardly we can see that the results from the paper continue to apply, as costlier wars (which increase the bargaining surplus) continue to create costlier peace by establishing greater incentive to exert diplomatic effort, and the expected aggregate payoff in peace continues to be eroded by steeper competition such that in the limit it is equal to the expected aggregate payoff in war.

Additionally, because zero is necessarily an accumulation point, it must be the case that there continues to be a nonzero risk of delay in the equilibrium of any game with settlement frictions $\mu > 0$. This, however, relies on non-degenerate mixed strategies, and hence $d > 2$.

C.2 Competitive Diplomacy with Flexible Recognition Probabilities

In this section, I allow the recognition probabilities to vary by country and state. A country i exerting greater effort than their opponent, $e_i > e_{-i}$, in state $s \in S$ results in country i being recognized as the agenda setter with probability $\pi_i(s)$. Specifically, these can vary across countries such that $\pi_i(s) \neq \pi_{-i}(s)$ as well as across states such that $\pi_i(s) \neq \pi_{-i}(s')$ for $s \neq s'$. One reasonable setting would be that $\pi_i(s)$ is increasing in a country i's relative strength, although I impose no restrictions of this kind. To demonstrate the core similarities and differences, I focus on the case without frictions $\mu = 0$.

A natural expectation would be that a country's incentive to exert effort in competitive diplomacy depends on the advantage they have in the competition. On the contrary, I find that there is a unique equilibrium in which countries play the identical strategies. This is because country incentives to compete are determined by their net gain from being the greatest performer, which is proportional to $\pi_i(s) - (1 - \pi_{-i}(s)) = \pi_{-i}(s) - (1 - \pi_i(s))$ for both countries given any couple of $(\pi_i(s), \pi_{-i}(s))$. The following proposition characterizes the equilibrium.

Proposition A5 (Equilibrium with Flexible Recognition)**.** *There is a unique equilibrium to a flexible recognition game without frictions where, for all states s and transitions q, each country $i = 1, 2$ plays $\sigma_i^{rec}(s) = (a_i^*(s), e_i^{rec}(s), x_i^*(s), y_i^*(x; s))$ defined as follows.*

(i) Decisions $a_i^*(s)$, $x_i^*(s)$, and $y_i^*(x; s)$ are defined in Proposition A1.

(ii) Effort $e_i^{rec}(s)$ is drawn from the cumulative distribution

$$F_s^{rec}(e) = \frac{(1-\delta\lambda)e}{(\pi_1(s)+\pi_2(s)-1)B(s)}$$

with $F_s^{rec}(e) = 0$ for $e < 0$ and $F_s^{rec}(e) = 1$ for $e > \frac{(\pi_1(s)+\pi_2(s)-1)B(s)}{1-\delta\lambda}$ if $U_i(s) \geq W_i(s)$. Otherwise exert zero effort $e_i^{rec}(s) = 0$.

Proof of Proposition A5. The proof follows the structure of the proof of Proposition A1 with the imposition that $\mu = 0$. The derivation of $a^*(s)$, $x^*(s)$, and $y^*(x;s)$ are identical to the derivation from Proposition A1. Now, the expected net gain to country i from exerting effort $e \geq 0$ on diplomacy is

$$\frac{B(s)}{1-\delta\lambda}\Big[\pi_i(s)\Pr(e > e_{-i}) + (1-\pi_{-i}(s))\Pr(e_{-i} > e)\Big] - e. \tag{A27}$$

The net gain is zero when $B(s) = 0$, in which case neither country will be willing to spend a positive amount in equilibrium. Therefore, to understand strategies with nonzero amounts of effort, assume $B(s) > 0$.

There are still no pure strategies under the identical logic as before. I now look for a mixed strategy corresponding to state s given by c.d.f. F_s^{rec} that satisfies equation (A27) for both countries.

We can write the payoff from zero effort as

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}(1-\pi_{-i}(s)). \tag{A28}$$

On the other hand, exerting effort of $e > 0$ yields an expected payoff

$$W_i(s) + \frac{B(s)}{1-\delta\lambda}\Big[F_s^{rec}(e)\pi_i(s) + (1-F_s^{rec}(e))(1-\pi_{-i}(s))\Big] - e. \tag{A29}$$

Using equations (A28) and (A29), we can solve for

$$F_s^{rec}(e) = \frac{(1-\delta\lambda)e}{(\pi_1(s)+\pi_2(s)-1)B(s)}. \tag{A30}$$

which implies an upper bound on equilibrium effort of $\bar{e}_s = \frac{(\pi_1(s)+\pi_2(s)-1)B(s)}{1-\delta\lambda}$.

Uniqueness follows from the same logic as Proposition A2. Consider the case where a country exerts effort $e > \bar{e}_s$ with nonzero probability. If both countries are, there is a profitable deviation to zero effort. If only one country is, by deviating back to $\bar{e}_s$, their payoff will be strictly more as they continue to gain only $W_i(s) + \frac{\pi_i(s)B(s)}{1-\delta\lambda}$ but expend strictly less effort. Further, consider a strategy where zero effort is expended with nonzero probability. Then, the country must be indifferent between expending zero and expending nonzero amounts of effort. However, this would imply the country is always the worst performer in equilibrium, in which case there is a strictly profitable deviation to never expending effort, which cannot be the case in equilibrium.

Next, consider a strategy where there is an atom on a nonzero amount of effort. Then, there is a strictly positive deviation to marginal levels of effort that disproportionately increase their expected win probability. Lastly, to show that zero must be the lower bound of the support, suppose otherwise that one country i always exerts at least effort $e' > 0$. Then, their opponent $-i$ will never exert amounts between zero and e' as exerting zero is strictly better. However, this implies that country i has a strictly profitable deviation to reducing effort levels below e' without altering the probability of being the stronger performer.

As before, equilibrium offers are given by equations (A8) and (A9), and a country receiving an offer rejects offers in their rejection set and accepts all others. □

The result is especially interesting because both countries play the same mixed strategy in equilibrium, which is counterintuitive given they have different absolute values for being the greatest performer. This is in contrast to other contests with different valuations (e.g., Hillman and Riley (1989)), where countries with a lower valuation tend to exert less effort on average. The reason for the difference is that, while their absolute values for being the greatest performer is different, these are offset by their differing absolute values for being the weakest performer. Consequently, regardless of how much larger one country's advantage is over the other, their expected net gain from becoming the agenda setter is equal to $\pi_1(s) + \pi_2(s) - 1$ times the expected value of recovering the bargaining surplus in the current period.

Appendix References

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