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Preventive war and sovereign debt

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Abstract

The preventive motive for war arises because states cannot commit to limit the use of their growing power. This commitment problem can lead to war when there are not enough resources available to compensate the declining state for their expected losses. In this article, we show how capital markets affect preventive war incentives by introducing a profit-maximizing bond market to the canonical bargaining model of war. We find that the nature of the power shift and fundamentals of the market for debt interact to determine when a preventive motive is more likely to lead to war. Two main results show that (1) less probable but more extreme power shifts are most dangerous and (2) unlike the direct effect of interest rates on the cost of war, higher interest on sovereign debt makes war more likely. We present evidence for the latter effect by extending Lemke's (2003) study of preventive war for major-power dyads between 1816 and 1992.

Keywords

Commitment problem, debt, game theory, war

A state that expects to experience a large shift in its military power faces a problem with its rivals. When a rising state grows stronger, it will demand more resources or a larger say in

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policy outcomes. The state in relative decline then faces a problem: the rising state cannot be expected to honor previous agreements that do not reflect their newfound power. Furthermore, a rising state can only freely transfer their current resources to the declining state in an attempt to stop a preventive war. If such a sacrifice is insufficient to compensate for the series of impending losses, the declining state may launch a preventive war. Here, war gives the declining state a chance to avoid the consequences of a power shift and to lock in their long-run access to resources.

At their core, such preventive wars result from a liquidity problem. The rising state does not have enough resources on hand to buy peace, but it is not insolvent and could credibly make transfers today if it could borrow against future streams of resources.[1]

To demonstrate how significant borrowing can be, consider the following example of a resource consumption problem between two states. Suppose that, in each period, two countries are sharing 1 unit of resources between them. Equating each period to a year, suppose the states have a 0.95 subjective discount rate of future periods. Assume the cost of war is 0.5, or half of all the resources available in a period. After a shift in power, suppose that the rising power will win a war against the declining power with 0.5 probability. By going to war after the shift is complete, the rising power can, with probability 0.5, secure for itself the present value of the entire pie in every future period after paying the cost of war. This value is approximately 9 pies in present value. If the rising state could borrow against this future value, then this is a source of transfers that is nine times the size of today's resources. That is, the traditional model only captures a tenth of the resources potentially available for bargaining. Of course, this is just an illustrative example, but there is nothing special about it. In fact, we can easily imagine realistic scenarios with far more extreme outcomes.

If credit market access determines a considerable portion of the resources available for bargaining, this fundamentally alters our understanding of preventive war, bargaining, and the connections between conflict and international finance. To explore the effects of borrowing on war, we build a simple model of preventive war where sovereign lending is possible. A rising state may sell bonds to a profit-maximizing market to generate resources that can then be transferred to a declining state. Our model considers a bond market where states are allowed to default, but will not as long as they have enough resources to pay their debt. We also simplify away from traditional consumption smoothing motives for state borrowing, choosing instead to focus on how borrowing may help states avoid preventive war owing to commitment problems stemming from stochastic shifts in power.

We find that borrowing from an outside lender or market against future gains can allow the rising state to avoid war. Still, preventive war is unavoidable in some cases. There are two reasons for this. First, states must pay interest on their loans; depending on market conditions, this interest may be so high as to prevent states from credibly borrowing enough to fully alleviate the threat of preventive war. Second, states only achieve gains in power with some probability. When power shifts fail to occur, states may default. If this risk of default is significant enough, lenders and states may be unwilling to agree to loans that could avoid preventive war. This means that both the nature of the power shift and the general market conditions can determine when commitment problems cause war.

In addition, three empirical predictions naturally arise from our simple setup. First, for future shifts of the same expected size, less probable but more extreme shifts are especially dangerous.[2] Markets are less able to provide liquidity given the high probability of default associated with extreme but uncertain power shifts. This indicates that preventive war will be more likely in situations where low-probability, high-impact changes are expected. For

example, a nuclear weapons program delivers a relatively low probability of success in any given period, but its enormous potential impact makes the commitment problems difficult to resolve through borrowing.

Second, even when states can successfully borrow against uncertain future power shifts, they will often pay a premium on their debt. Markets will demand higher rates in order to cover themselves in the event that the state fails to grow more powerful and is forced to default. This effect may address why rapidly growing states in adverse security environments, like South Korea, pay a premium on their debt vs states in more benign security situations.[3]

Third, all else equal, when the global real risk-free interest rate is high, war is more likely through the commitment problem mechanism. In our model, the higher the risk-free rate, the better the bond market's outside option is relative to lending to a rising power. This makes bondholders less willing to lend, while loans that do occur are burdensome. Under these circumstances, potential borrowers may even prefer risking war to the high-rate loans they are offered.[4] Moreover, while the risk-free rate is exogenous in our model, it allows us to draw a direct connection between events like the Great Depression, which increased the cost of capital, and the subsequently heightened dangers of war owing to commitment problems. Working with this example, Romer (1992) notes that real interest rates in the USA skyrocketed in the early part of the Great Depression, and then again in 1937. Additionally, US lending to Europe dropped by more than 75% from 598 million dollars in 1928 to 142 million dollars in 1929 (Kindleberger, 1973: 56). The economic implications were particularly serious for Germany, which relied on US loans to make reparations payments.[5] After the theoretical analysis, we will explore this last empirical implication by extending Lemke's (2003) analysis to include international interest rates. We find initial support for this prediction.[6]

Commitment problems and sovereign debt

This analysis is related to two prominent ideas in international relations: commitment problems and sovereign default.

The international relations literature on commitment problems in crisis bargaining begins with Fearon (1995) and is theoretically developed in subsequent papers (Bas and Coe 2012; Chassang and Padró i Miquel, 2019; Debs and Monteiro, 2014; Fearon, 1996, 2004; Krainin and Wiseman, 2016; Krainin, 2017; Krainin and Slinkman, 2017; Leventoglu and Slantchev, 2007; Powell, 1999, 2004, 2006, 2012, 2013; Wiseman, 2017).[7] Commitment problem models have recently been utilized to understand a number of applied issues, including civil wars (Paine, 2016) and the interactions between domestic politics and the potential for interstate war (Chapman et al., 2015). Moreover, new techniques have been developed to test commitment problem models empirically (Bas and Schub, 2017; Bell and Johnson, 2015; Lemke, 2003). With all this, however, no paper has yet addressed the impact that borrowing against the future may have on the liquidity constraint that lies at the heart of the commitment problem.

To address this, we connect the international relations literature on commitment problems to the economics literature on sovereign default. The default side of our model is most closely connected to the one developed in Arellano (2008). Eaton and Gersovitz (1981) provide a classic contribution to this literature while Chatterjee et al. (2007) make important

recent theoretical advances in the context of strategic consumer default. The literature on sovereign default includes a vast number of papers that make theoretical and empirical contributions. Papers in this literature, however, do not model lending in a strategic security context. Therefore, they cannot address how the specter of preventive war may lead to debt buildups and subsequent defaults.

Some recent literature has focused on debt financing war efforts. McDonald (2011) demonstrates how sovereign lending allows states to maintain arms races without having to renegotiate their society's basic social contract.[8] Slantchev (2012) builds a model where states may borrow unlimited amounts of debt to finance mobilization efforts and default occurs upon defeat. Slantchev establishes that the incentives which states have to borrow can endogenously induce conflict. In his model, borrowing can endogenously increase the cost of preserving a peaceful status quo relative to war because war lowers the burden of debt by allowing the defeated state to default.[9] In contrast to works that emphasize war as a driver of sovereign default, Shea and Poast (2017) find that states are unlikely to default after losing a war. Their reasoning aligns with an effect present in our model—lenders will strategically limit loans to amounts that will, in all likelihood, be paid back. Finally, Poast (2015) notes how states that possess central banks are better able to secure debt financing, especially in times of war.

Another recent publication has focused on war finance directly. Zielinski (2016) links the severity of a war's consequences to the manner in which they are financed, which is determined in part by state capacity and the preferences of leaders. She argues that war durations are affected by the interaction between the conflict and citizens. Through a proposed "war finance continuum," Zielinski places sovereign debt low on citizen consciousness, as opposed to taxation and other, more direct resource extraction methods.

A long-running literature has argued that financial interests work to create peace. Polanyi (1944) argues that powerful, cartel-like financial interests actively pushed the international system toward peace in the 19th century. Recently, Flandreau and Flores (2012) have refined Polanyi's argument, suggesting that "prestigious" financial certification intermediaries act to avoid war in order to avoid the possibility of sovereign default and the consequent damage to their reputations. Alternatively, Kirshner (2007) proposes a preference-based argument emphasizing that financial communities are averse to war due to its deleterious impact on macroeconomic stability.

In our model, financial interests help states avoid preventive war. Their actions, however, are a direct consequence of their profit-maximizing motivation. Purely through their pursuit of the most profitable investments, financial interests may help states maintain peaceful international bargains. One advantage of our analysis is that the limits of this incentive are clear, and we are therefore able to identify when wars will occur in spite of financial interests. Moreover, our argument does not preclude the peaceful effects others have pointed to in this literature, which may also motivate financiers beyond profit.

Our setup is both simple and abstract. We consider a pie, a discount bond sale, a transfer, and a repayment. In some cases, such as German war reparations, the straight line between the debt instrument and the transfer is clear, but in most cases it is not so direct. States participate in many activities requiring spending and bond issuances, complicating this direct causal link. Moreover, different kinds of policy choices can be essentially equivalent to transfers. As a result, our theory provides a clear understanding of the bounds on how credit markets affect wars resulting from commitment problems. The relevant empirical implications of the relationship between central variables of our theory and key measures of international

conflict are indirect. For example, we don't mean to imply that China will issue 10 year bonds and wire the cash to the US Treasury, but we do expect changes in the risk-free rate to have wide-ranging effects on conflicts with preventive war incentives. That said, we can show that interest rates significantly affect war onset by increasing the presence of the preventive motive.

Another important consideration is the scope conditions for this mechanism. First, we expect the effect of interest rates to apply when power shifts and commitment problems dominate strategic decision-making, but not necessarily when asymmetric information—either about power or resolve—is the primary strategic factor. Second, the entire bargaining framework assumes that the issue is in some way divisible or that a compromising policy is feasible. Our model further assumes that side-payments make sense in this context. For example, even if one were able to find a way to "share" an indivisible sacred space, transfers or side-payments might not be politically acceptable as an alternative and, in some cases, might be considered blasphemy.[10]

Model

To model the bargaining problem with power shifts and a financial market, we start with the canonical bargaining model of war and add a profit-maximizing bond market. As we will see, the bond market has important effects on the probability of conflict emerging from the commitment problem.

Players and resources

There are two states, Home (H) and Foreign (F), that interact over two periods, $t \in \{1, 2\}$. H can borrow against the future through the bond market via Lender (L) for an amount that it can use in bargaining with *F*oreign.[11] Namely, in period 1, H can sell one-period discount bonds B at a "discount price" $q<1$. The countries must bargain over an international flow of benefits each period, normalized to size 1, plus qB if *H*ome chooses to borrow, knowing they will need to pay back B in the future. *F*oreign makes a take-it-or-leave-it proposal x_t in each of two periods, where *H*ome receives x_t and *F*oreign receives the remainder of the benefits plus any amount borrowed by H.

Future periods are discounted at the common rate $\beta \in (0, 1)$. We think of period 2 as representing the entire future; therefore, payoffs in period 2 are valued at $\beta/(1-\beta)$ times the value of payoffs in period 1. Hence, H's total utility for a peaceful sequence of bargain offers is

$$x_1 + \frac{\beta}{1-\beta} x_2,$$

while *F*oreign's total peaceful utility without borrowing is

$$(1 - x_1) + \frac{\beta}{1-\beta}(1 - x_2).$$

In the case where H borrows from L, *F*oreign gains an additional one-time transfer of qB.

The bond market

We start by considering a financial market consisting of profit-maximizing traders, collectively acting as the non-strategic lender L, who are willing to buy bonds from *H*ome. L can alternatively lend money at an international interest rate $r > 0$. If feasible, *H*ome commits to pay back L for any borrowings. Therefore, *H*ome does not default strategically and only defaults when it lacks the ability to pay back its bonds. This happens with probability δ, a value that will be determined endogenously and described in a subsequent section. Other than maximizing its return, L has no further interest in outcomes for *H*ome and *F*oreign. We also assume that *H*ome places a sufficient value on the future,

$$\beta > \frac{1}{1+r}$$

so that it does not have an incentive to borrow against future wealth in order to consume more today purely owing to impatience. For simplicity, we also do not allow other states, including *F*oreign, to buy *H*ome's bonds.[12]

Our commitment assumption that *H*ome does not default strategically is a stark simplification, but could be justified in two ways in a more general model. One, *H*ome may want to borrow for a variety of reasons (such as consumption smoothing) and loses access to markets after defaulting. Two, the model could be extended to consider repeated shifts in power so that *H*ome must preserve a good reputation with bond traders to preserve liquidity in case of future shifts.

Empirically, countries borrow for many reasons and bond income goes into general funds that are used for many things—building roads, social welfare programs, foreign aid, etc. Some of these programs can be considered an example of borrowing in the name of growing power. Take China's Belt and Road Initiative: since the 2013 announcement of One Belt and One Road, the American Enterprise Institute's China Global Investment Tracker has recorded $1.425 trillion in Chinese investment abroad (Scissors, n.d.). This has more than doubled China's spending from 2005 to 2012 and is double the direct costs of the US involvement in the Iraq War (Crawford, 2020). Between 2013 and 2017, the most recent year for which there is quality data, China's public debt-to-GDP ratio has gone from 16.26 to 35.06%, even as real economic growth has been strong (World Bank, 2021). All of this debt is not solely related to the Belt and Road, but the logic of this influence program is to integrate and ingratiate China with both regional and major world powers.

Nevertheless, we emphasize that, while our model considers the single war-oriented motivation for borrowing, its real world implication is more an association between the price of public debt and peace than accumulations of public debt by itself.[13]

War

In our model, war is represented as a costly lottery. Each state wins the war with some probability and pays costs $\kappa > 0$. The winning state captures the value of the international flow of benefits in both periods. The losing state can still consume any domestic resources, but can no longer challenge the winning state for a portion of the international pie. We can consider the losing state disarmed.

Table 1. Notation.

Home country (player)	H
Foreign country (player)	F
Non-strategic lender (bond market)	L
Time period	t
Take-it-or-leave-it offer in period t	x_t
Bond amount	B
Smallest bond necessary to prevent war	B^*
Discount price on the bond	q
Common discount rate for future periods	β
International interest rate	r
Probability of default	δ
Cost of war	κ
H's probability of victory (strength)	s
Probability of exogenous power shift	ρ
Shock to H's probability of victory (military capabilities)	θ

In this model, states win a war with an exogenously determined probability. In period 1, H's probability of winning a war, or strength, is s and Foreign's probability of victory is $1 - s$. If war occurs in period 1, then the value of war to H is

$$\frac{1}{1-\beta}s - \kappa.$$

We assume that this value is positive to avoid uninteresting cases.

In the event that H wins the war, H captures the entire international pie of size 1 today and in the future. This is multiplied by $1/(1 - \beta)$ to account for period 2 representing the entire future. Foreign's value for war in period 1 and the value of war for both states in period 2 can be similarly defined.

Exogenous power shifts

Now consider the impact of a potential exogenous power shift, which occurs at the end of period 1 with probability $\rho \in (0, 1)$. If no shift occurs, H's probability of victory remains the same as in period 1, at s. If a shock does occur, H's probability of victory increases to θs, with[14]

$$\theta \in \left(1, \ \frac{1}{s}\right).$$

For reference, all our notation is collected in Table 1.

Timing

Putting it all together, period 1 proceeds as follows:

1. Home and Foreign both learn the values of ρ and θ.
2. Home chooses how much to borrow, qB, this period.

3. *F*oreign makes a take-it-our-leave-it offer to *H*ome of x_1, wherein *F*oreign either gets the remainder of the international pie, $1 - x_1$, as well as whatever *H*ome borrowed or declares war.
4. *H*ome accepts or rejects the offer. If *H*ome accepts, the states peacefully consume their allocations. If *H*ome rejects, war occurs and the states receive their war payoffs.
5. Power shift θ occurs with probability ρ.

Period 2 proceeds in the same way except that steps 1, 2, and 5 are skipped and that, if war has occurred, the winner receives the whole international pie. Our solution concept is subgame perfect equilibrium (SPE).

Analysis

Power shifts without borrowing

In this portion of the analysis, we explore how *H*ome behaves when borrowing is not a possibility. We can see how our problem relates to the classic explorations of the commitment problem in Fearon (1995, 2004) and Powell (1999, 2004, 2006). This relationship is easiest to see when $\rho = 1$, guaranteeing that a power shift will occur. When this is the case, H has an initial war value in period 1 of

$$\frac{1}{1-\beta}s - \kappa,$$

which, in period 2, increases to

$$\frac{1}{1-\beta}\theta s - \kappa$$

after the power shift takes place.

Anticipating this shift in power, *F*oreign prefers war to any bargain when its period 1 war value is greater than the largest bargain H can credibly commit to in the future. This amount is the entire pie today plus the entire future bargain value less the discounted value of *H*ome's period 2 war value. That is,

$$\frac{1}{1-\beta}(1-s) - \kappa > 1 + \frac{\beta}{1-\beta} - \beta\left[\frac{1}{1-\beta}\theta s - \kappa\right].$$

After some rearrangement, we can solve for the minimum θ that leads to war. This result is presented as Lemma 1.[15]

Lemma 1. *When there is no borrowing and* $\rho = 1$*, then war occurs if and only if*

$$\theta > \frac{1-\beta}{\beta s}(1+\beta)\kappa + \frac{1}{\beta}.$$

This result is essentially identical to the conditions on war found previously in the literature for one-period exogenous shifts in power (Powell, 2006). The size of the shift necessary

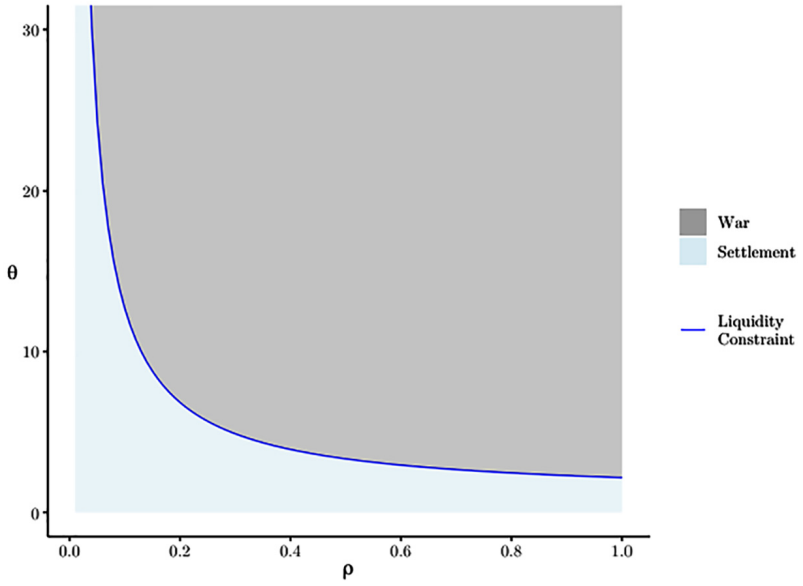

Figure 1. War with uncertain power shifts.
The *x*-axis is the probability of a power shift ρ and the *y*-axis is the value of shocks θ. A point in the plane is a transition probability–power shift pair. The blue line marks the transition between the case where war occurs in the standard model, to the north and east.

to cause war is increasing in the costs of war, but decreasing in the discount rate and in the initial probability of victory for the rising state.

The first modification from the canonical case that we make is to allow for the possibility that $\rho \neq 1$. Under this condition, a shift like the one above will occur with probability ρ, while a shift will not occur with probability $1-\rho$. This leads to a modification of the condition in Lemma 1, which is stated in Lemma 2.

Lemma 2. *When there is no borrowing, then war occurs if and only if*

$$\theta > 1 + \frac{1-\beta}{\beta\rho}\left(1 + \frac{1+\beta}{s}\kappa\right). \tag{1}$$

As the probability of the shift (ρ) goes to one, smaller changes in *H*ome's military capabilities (θ) can lead to war as a result of the commitment problem. Figure 1 illustrates this. The condition for war with a probabilistic shift otherwise behaves in the same manner as when the shift is certain, with the minimum shift size increasing in the costs of war and decreasing in the discount rate and the initial probability of victory.

The effect of borrowing

The condition from Lemma 2 is represented by the gray-shaded region of Figure 1, defining the circumstances where *H*ome may choose to borrow from the bond market. *H*ome only

wants to borrow in order to avoid war and, therefore, only borrows when inequality (1) is satisfied. There are two further conditions on borrowing explored in detail below. First, the buyer of the bond must prefer lending *H*ome a sufficient amount of money to avoid war to its outside option for that money. Here, that outside option is the exogenously given global risk-free interest rate, r. Second, *H*ome must prefer borrowing enough to avoid war to the payoff from war.[16]

Borrowing increases the possibility of peace by transferring money from *H*ome's future income to the present, where it can be credibly transferred to *F*oreign. The amount of borrowing necessary to prevent war can be derived directly from inequality (1), since *F*oreign's utility from fighting in period 1 must be at least as much as its utility from getting all of the pie in period 1, its portion of the pie in period 2, and any amount *H*ome borrows to appease *F*oreign. Therefore, borrowing amount qB today is just sufficient to prevent war when

$$\frac{1}{1-\beta}(1-s)-\kappa = 1+\frac{\beta}{1-\beta}+\beta\kappa-\rho\left(\frac{\beta}{1-\beta}\theta s\right)-(1-\rho)\left(\frac{\beta}{1-\beta}s\right)+qB^*.$$

We can solve for the minimum loan qB^* required to prevent war. The result is presented in Lemma 3.[17]

Lemma 3. *When there is borrowing, F requires at least*

$$qB^* = \frac{s\beta\rho}{1-\beta}(\theta-1)-s\left(1+\frac{1+\beta}{s}\kappa\right)$$

to opt for peace.

*H*ome has to borrow more when the probability of a power shift is higher and when the size of a possible shift increases since *F*oreign will be more worried about the consequences of a power transition. On the other hand, *H*ome borrows less when the cost of war is higher since *F*oreign is less worried about its willingness to fight wars. The effect of the discount rate and the initial probability of victory are ambiguous on the borrowed amount.

The bond market's perspective. While the incentives to borrow and the conditions for peace between *H*ome and *F*oreign are straightforward, the bond market's perspective is more nuanced. *H*ome defaults on its loans in period 1 when it lacks the resources to pay back bondholders. We use δ to denote the probability of default (this will be determined endogenously later). Before calculating the probability of default for various parameters, first consider the *L*ender's bond purchasing decision. By borrowing from *L*, *H*ome promises to pay B back to bondholders in period 2 while receiving qB today. *H*ome, however, can only pay *L* back with endogenous probability $1-\delta$. Alternatively, market actors could lend qB on the international market and receive back $(1+r)qB$ next period with certainty. By no arbitrage, the expected return must be equivalent under both investment schemes, so that

$$(1-\delta)B = (1+r)qB$$

$$q = \frac{1-\delta}{1+r}.$$

Hence, we can calculate the price of the bond q as a function of the default rate δ and the going risk-free rate r.[18]

Now consider *H*ome's borrowing decision. *H*ome pays a premium on borrowed money and, therefore, only borrows either when q is low relative to future value of consumption—that is, when *H*ome would prefer to consume tomorrow's income today because interest rates are low and it is impatient—or when liquidity is constrained, so that transfers are needed in order to avoid war.[19] Our model rules out the first possibility with the assumption that $\beta > 1/(1+r)$. Therefore, *H*ome does not borrow when it can buy peace without borrowing, which happens when inequality (1) does not hold.

When inequality (1) does hold, *H*ome prefers to borrow and avoid war so long as borrowing and avoiding war give a higher payoff than fighting. In order to calculate this, we must first calculate the value of borrowing. If borrowing is sufficient to prevent war in the initial period, two things may happen in period 2. Either *H*ome experiences a positive shift and can thus borrow and pay back up to the entire present value of future payoffs determined by *H*'s war value, in this case

$$\beta\left(\frac{1}{1-\beta}\theta s-\kappa\right),$$

or the shift does not happen and H can only pay bondholders back with

$$\beta\left(\frac{1}{1-\beta}s-\kappa\right).$$

As determined above, B^* is the smallest bond necessary to prevent war and has a bond price

$$qB^*=\frac{1-\delta}{1+r}B^*$$

Therefore, there exist three borrowing regions. In the region where

$$B^*>\beta\left(\frac{1}{1-\beta}\theta s-\kappa\right),$$

Home cannot commit to ever repaying and default is assured, so $\delta = 1$. In the region where

$$B^*<\beta\left(\frac{1}{1-\beta}s-\kappa\right),$$

then $\delta = 0$ and default never occurs. If B^* is between these two values, then δ is equal to the probability of no shift, or $1-\rho$. These regions imply a q of 0, $1/(1+r)$, and $\rho/(1+r)$, respectively. The bond market will only lend to *H*ome in the latter two cases.

The borrower's perspective. Although borrowing can reduce the likelihood of war, war may still occur if the *L*ender will not loan as much as *H*ome needs or if *H*ome does not think borrowing is a better deal than going to war. First, consider solutions where $\delta = 0$, which is only the case when

$$\frac{1}{1+r}B^*=\frac{s\beta\rho}{1-\beta}(\theta-1)-s\left(1+\frac{1+\beta}{s}\kappa\right)$$

and

$$B^* < \beta\left(\frac{1}{1-\beta}s - \kappa\right).$$

The bond market will not offer such a loan when

$$s(1+r)\left[\frac{\beta\rho(\theta-1)}{1-\beta} - \left(1 + \frac{1+\beta}{s}\kappa\right)\right] > \beta\left(\frac{1}{1-\beta}s - \kappa\right).$$

In any situation where borrowing is considered, *H*ome cannot appease *F*oreign in period 1 simply with the existing international pie, so it will either go to war in the first period or it will give up all of the pie and a borrowed amount in the first period while gaining part of the pie in the second period. Substituting the above value of B^*, this inequality indicates that war is still preferred to no-default borrowing when

$$\frac{1}{1-\beta}s - \kappa > \beta \times \left[\rho\left(\frac{1}{1-\beta}\theta s - \kappa\right) + (1-\rho)\left(\frac{1}{1-\beta}s - \kappa\right) - s(1+r)\left[\frac{\beta\rho(\theta-1)}{1-\beta} - \left(1 + \frac{1+\beta}{s}\kappa\right)\right]\right].$$

Analogously, a risky loan with $\delta = 1 - \rho$ gives the bond amount

$$\frac{\rho}{1+r}B^* = \frac{s\beta\rho}{1-\beta}(\theta - 1) - s\left(1 + \frac{1+\beta}{s}\kappa\right),$$

such that

$$\beta\left(\frac{1}{1-\beta}s - \kappa\right) < B^* < \beta\left(\frac{1}{1-\beta}\theta s - \kappa\right).$$

In other words, the bond market will still not lend if

$$\frac{s}{\rho}(1+r)\left[\frac{\beta\rho(\theta-1)}{1-\beta} - \left(1 + \frac{1+\beta}{s}\kappa\right)\right] > \beta\left(\frac{1}{1-\beta}\theta s - \kappa\right).$$

Here, there is a $1 - \rho$ chance of default, in which case *H*ome does not have to pay off the loan, but the effect on *H*ome's finances is canceled out by the difference in pricing from $q = [1/(1+r)]$ to $q = [\rho/(1+r)]$. Therefore, *H*ome prefers war to either kind of borrowing when

$$\frac{1}{1-\beta}s - \kappa > \beta \times \left[\rho\left(\frac{1}{1-\beta}\theta s - \kappa\right) + (1-\rho)\left(\frac{1}{1-\beta}s - \kappa\right) - \rho\left(\frac{s}{\rho}\right)(1+r)\left[\frac{\beta\rho(\theta-1)}{1-\beta} - \left(1 + \frac{1+\beta}{s}\kappa\right)\right]\right].$$

These conditions are summarized graphically in Figure 2. War occurs if inequality (1) holds so that *F*oreign prefers war to peace without borrowing and *H*ome either prefers war to

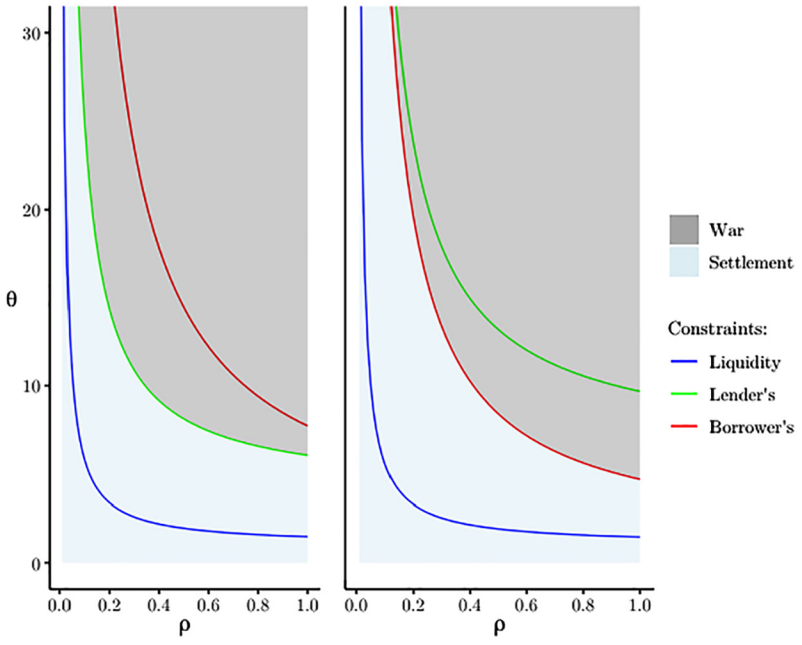

Figure 2. Peace bonds and the incentives to borrow and lend.
The *x*-axis is the probability of a power shift ρ and the *y*-axis is the value of shocks θ. A point in the plane is a transition probability–power shift pair. The blue line marks the transition between the case where war occurs in the standard model, to the north and east, and where there is a settlement as in Figure 1. The green and red curves represent the lender's and the borrower's constraints. The gap between the lower of these two curves and the blue curve marks the increase in the region of peace.

the amount of borrowing that prevents it (condition 2), or the bond market is not willing to offer a loan for the amount *H*ome needs (condition 3). War then occurs when either *L*'s lending constraint or *H*ome's borrowing constraint is violated. Clearly, access to credit markets still leaves room for war. We express these latter two conditions in Lemmas 4 and 5, where the condition for *H*ome choosing to fight rather than borrow is rearranged and simplified.

Lemma 4. *Home prefers fighting to borrowing when*

$$\theta\rho > \rho + \frac{1-\beta}{\beta} - \frac{\kappa}{s}\left(\beta + \frac{1-\beta(1-r)}{\beta[1-\beta(1+r)]}\right). \tag{2}$$

Lemma 5. *The bond market will not offer a loan if*

$$\frac{s}{\rho}(1+r)\left[\frac{\beta\rho(\theta-1)}{1-\beta} - \left(1+\frac{1+\beta}{s}\kappa\right)\right] > \beta\left(\frac{1}{1-\beta}\theta s - \kappa\right). \tag{3}$$

We analyze the comparative statics of inequalities (1)–(3) from Lemmas 2, 4, and 5, respectively, in the following section.

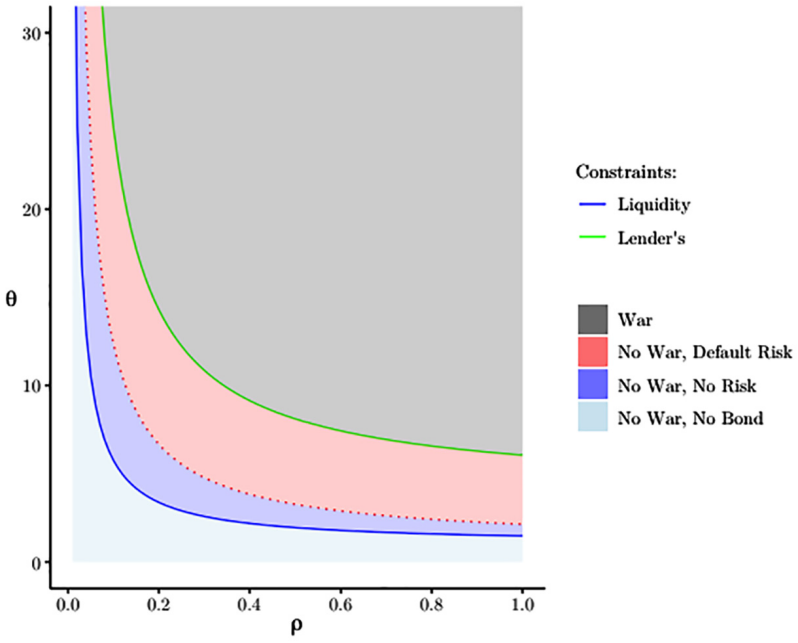

Figure 3. Borrowing, war, and the risk of default.
The *x*-axis is the probability of a power shift ρ and the *y*-axis is the value of shocks θ. A point in the plane is a transition probability–power shift pair. The blue line marks the transition between the case where war occurs in the standard model, to the north and east, and where there is a settlement as in Figure 1. The green curve represents the lender's constraint. The gap between the green curve and the blue curve marks the increase in the region of peace, which includes the red area where default risk is incurred.

Figure 3 shows the different conditions one might encounter in a given crisis. The borrower's or the lender's constraint might bind, but we show the lender's constraint. As the example demonstrates, the market is often, but not always, willing to make risky loans that will prevent war.

Potential for conflict despite the possibility of borrowing

Recall inequalities (1)–(3). War occurs whenever (1) is satisfied and borrowing fails to occur. This can happen if either *H*ome prefers paying the cost of war to borrowing at a high premium, as in (2), or *L* will not lend because *H*ome would default at any market-driven interest rate, as in (3). Proposition 1 shows that increasing either the shift probability ρ or the size of the potential shift θ increases the likelihood of a preventive war.

Proposition 1. *There exist shifts that cause war even with borrowing. Increasing the expected size of a shift increases the potential for conflict.*

The proof follows fairly directly from the inequalities.[20] Qualitatively, this proposition suggests that borrowing does not upend our thinking about power shifts and that the basic

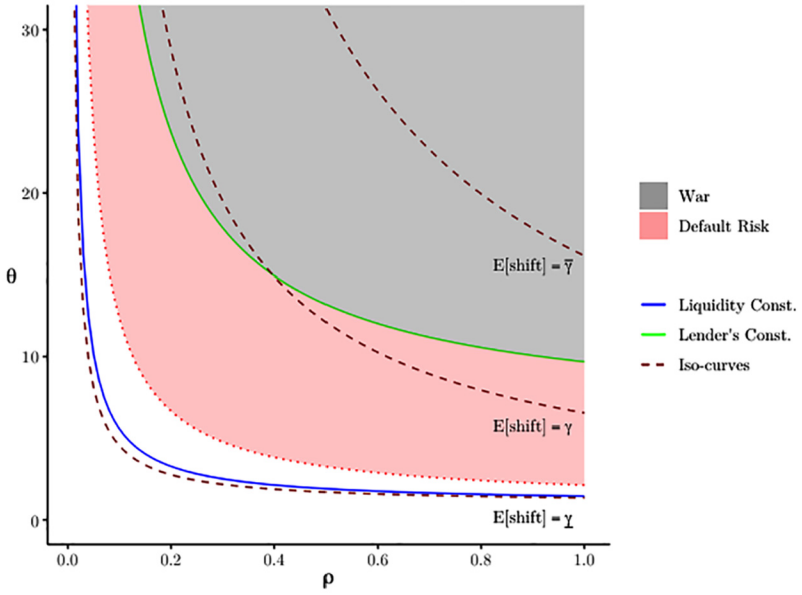

Figure 4. Mean preserving lotteries and solving the commitment problem through debt.
The *x*-axis is the probability of a power shift ρ and the *y*-axis is the value of shocks θ. A point in the plane is a transition probability–power shift pair. The blue line marks the transition between the case where war occurs in the standard model, to the north and east, and where there is a settlement as in Figure 1. The green and red curves represent the lender's and the borrower's constraints. The gap between the lower of these two curves and the blue curve marks the increase in the region of peace. The dashed lines are iso-lotteries that give the same expected value of a power shift, but different variance.

dynamics seen in the non-borrowing case still hold. As illustrated in Figure 3, borrowing does not prevent all wars as θ increases (and the argument for ρ is analogous). Another way to think about this proposition is that, if a power shift described by (θ, ρ) first-order stochastically dominates the shift (θ', ρ'), then (θ, ρ) has the higher potential for conflict. When shifts are more likely or more extreme, war becomes more likely. Even as $\beta \to 1$, some power shifts may cause war. Increasing β makes condition (1) easier to satisfy and both conditions (2) and (3) are always satisfied as $\beta \to 1$. The lower bound of constraint (2) converges to ρ, which is always less than the left-hand side value $\theta\rho$ since

$$\theta \in \left(1, \frac{1}{s}\right)$$

and $s<1$ are assumed. Likewise, condition (3) is always satisfied since the left-hand side goes to positive infinity while the right-hand side is finite.

We next look at instances where changes in the type of shift increase the potential for conflict by increasing the range of parameters for which war occurs. Two shifts, (θ, ρ) and (θ', ρ'), have the same expected size if

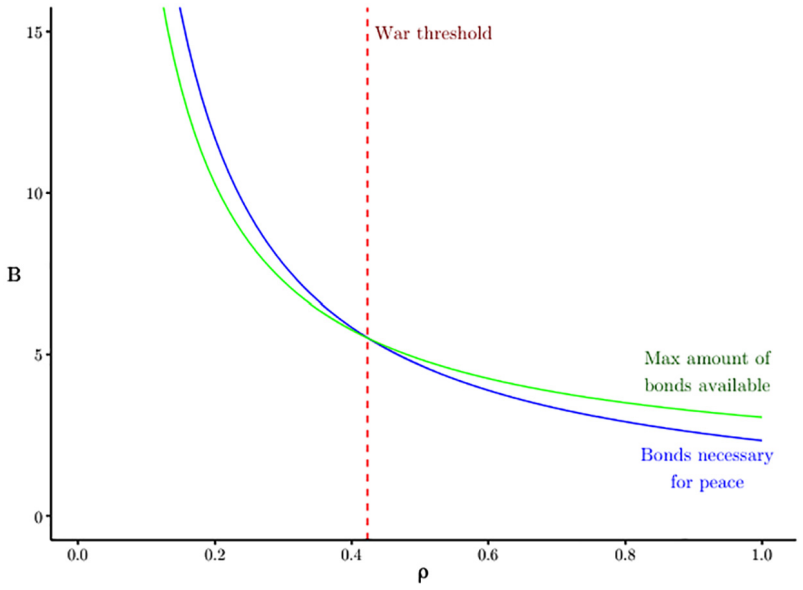

Figure 5. The war threshold.
Consider an initial shift of (θ, ρ). Proposition 2 states that if a shift (θ', ρ') is of the same expected size, then the more extreme/less likely (θ', ρ') is, the greater the potential for conflict. The figure above plots how the left-hand and right-hand sides of inequality (3) change in ρ'. The left-hand side (the blue line) indicates the necessary amount of bonds to avoid war. The right-hand side (the green line) indicates the maximum amount of bonds *L* is willing to buy. When the blue line is above the green line, war results.

$$\rho\left(\frac{\beta}{1-\beta}\theta s\right) + (1-\rho)\left(\frac{\beta}{1-\beta}s\right) = \rho'\left(\frac{\beta}{1-\beta}\theta' s\right) + (1-\rho')\left(\frac{\beta}{1-\beta}s\right)$$
$$\rho\theta s + s - \rho s = \rho'\theta' s + s - \rho' s$$
$$(\theta - 1)\rho s = (\theta' - 1)\rho' s.$$

Note that, if two shocks have the same expected size, then *F*oreign expects to offer *H*ome the same portion of the pie in the second period, so it can be bought off with the same transfer amount:

$$qB^* = q'B'.$$

Since traders on the bond market care as much about the likelihood of repayment as about the rate of repayment, they may refuse to lend in situations where *H*ome's expected power shift is extreme but improbable. Even when *H*ome and *F*oreign's calculations are not affected because there is no change in the expected size of the shift, *L* may prefer not to lend and leave *H*ome with no recourse but war.

Proposition 2. *For shifts of the same expected size, lower probability but more extreme shifts increase the potential for conflict.*

If a power shift (θ, ρ) second-order stochastically dominates a shift (θ', ρ'), then (θ', ρ') has a higher potential for conflict, even though the shift (θ', ρ') is a mean-preserving spread of shift (θ, ρ). This conclusion can be generalized to a continuous distribution of shifts.

Figure 4 illustrates this proposition for shifts that could induce risky borrowing. Shifts that are more extreme but lower probability are more unstable because it is harder for bond markets to provide the necessary liquidity. As you can see in the lower right of Figure 4, a shift of expected size γ can be compensated for by selling a risky bond, but a riskier shift of the same expected size leads to war. This has qualitative implications for interstate conflict. For example, this may explain differences in how countries respond to purchases of conventional military equipment (low θ, high ρ), as opposed to investments in nuclear weapons technology (high θ, low ρ). Proposition 2 suggests that the latter would more frequently cause conflict. Similar effects can be seen in Figure 5, which displays the bond amounts available and demanded for a given value of ρ.

Finally, H may not be able to borrow at rates that it can afford if L has safer lending opportunities elsewhere because r is high.

Proposition 3. *Increasing the world risk-free interest rate, r, increases the potential for preventive conflict.*

From this proposition, we conclude that exogenous shocks to the world economy that increase the cost of capital will also increase the likelihood of war. Such exogenous shocks could come in many forms. Burgeoning conflict in other parts of the world may cause increasing rates, which in turn cause bond traders to pick and choose where they lend. This could serve as a contagion channel for war to spread.

This result differs from the results one would expect for wars in general. As Zielinski (2016) and Rasler and Thompson (1983) note, among other aspects of finance, higher interest rates make borrowing to fight more and should generally decrease the likelihood of conflict.

Strategic lenders and the existence of preventive war

Our model contains a number of simplifying assumptions. It is reasonable to ask how relaxing some of these assumptions might impact the main result of the paper—namely, Proposition 1, which states that sovereign borrowing may alleviate, but not wholly eliminate, the possibility of preventive war. We make three key assumptions in our model: (1) full commitment to repay loans; (2) loans are "outside money"; and (3) loans are made by a "non-strategic" bond market.

To understand these assumptions clearly, first imagine reversing assumption (3).[21] There are two countries, *H*ome and *F*oreign (as in our baseline model), but no "non-strategic" bond market to provide loans. Instead, let the only source of loans for *H*ome be *F*oreign, who will certainly be strategic in its loan-making decisions. In this setting, *F*oreign may have an added incentive to provide loans to *H*ome beyond that which a non-strategic bond market is willing to supply, since *F*oreign internalizes the costs and risks of war while the bond market only cares about the return and risk on its loan. Having reversed (3), however, does it then make sense to maintain assumptions (1) and (2)?

The answer to both is "no." Assumption (2) is now nonsensical owing to physical constraints. The model is about two countries bargaining and fighting over the entire pie

between them. If there were some outside source of money that these countries have internal access to, then why can they not use this money for transfers directly? Why can this money be tapped for loans, but cannot be captured in war? In fact, if countries have access to unlimited internal funds, preventive war in the sense of Powell (2004) is immediately ruled out. If there is some limited source of internal funds, then it is unclear why these would not be included in the initial pie at issue between the two countries.

As for assumption (1), its maintenance in this setting would be equivalent to saying that *H*ome cannot commit to making transfers after a power shift, but can fully commit to paying back a loan. When dealing with an outside bond market, commitment could make sense for the reasons presented in the baseline model—maintaining access to the bond market will allow for future consumption smoothing and future loans to avoid war in the event of future power shifts. *H*ome may face some of these same incentives when receiving loans from *F*oreign. It is unclear, however, how *H*ome defaulting on promised future loan payments would be any different from defaulting on future transfers. To the extent that we assume *H*ome cannot commit to transfers to *F*oreign in the future, we must also assume that *H*ome cannot commit to future loan payments to *F*oreign.

Therefore, if we reverse assumption (3), we must also reverse assumptions (1) and (2). The natural thing to ask next would be: what if there are other strategically connected countries that may want to provide a loan to *H*ome for strategic reasons? This would constitute the lending of "inside money" in the sense that it is part of some country's resources within a strategically connected system.

To consider this situation, we must first clarify what it means for another country to be strategically connected to *H*ome and *F*oreign. One definition would be that this other country *i*'s security is strategically impacted by a war between *H*ome and *F*oreign. *H*ome or *F*oreign must be capable of going to war with *i* today or at some point in the future, possibly contingent on other conflicts. Then, *H*ome, *F*oreign, and *i* are in a connected network where there is some path by which conflict can spread from *H*ome and *F*oreign to *i*. For example, suppose there are three additional countries *A*, *B*, and *C*. Country *C* is connected to both *H*ome and *F*oreign on some path and is therefore strategically relevant. On the other hand, countries *A* and *B* are not strategically connected, although they might be connected through market changes that we discuss below. In this system, we can think of the bond market *L* as being an isolated actor who cannot be "attacked" or invaded by any other state. In this sense, *L* is providing money outside of strategic military considerations and is hence a source of what we call "outside money."

So, consider a situation in which a current development between *H*ome and *F*oreign has the potential for conflict, but *C* can use its resources to lend to *H*ome, preventing a war that might bring *H*ome to *C*'s doorstep. It is straightforward to show that there is no way for countries to transfer or loan each other "inside money" in a connected network of states in order to avoid war. This gives us Proposition 4, whose technical details can be found in the Supplemental Material.

Proposition 4. *There always exist values for power shifts, cost of war, and discount factors in the international system game such that preventive war occurs in a subgame perfect equilibrium when strategic lenders can loan each other inside money.*

What if, however, there exist both a connected network of strategically interested countries *and* some outside source of money (a group of unconnected or non-strategic countries)? Clearly, by Proposition 4, some amount of outside money will be necessary to maintain

peace for some power shifts. Regardless, we have demonstrated in this paper that the amount of outside money from a non-strategic source will always be limited by a combination of the going interest rate and endogenous default. Simply put, preventive war always exists in this environment. Moreover, shifts in power will be most dangerous in the same way as earlier—when they are greater in expected value, when r is large, and, for a fixed expected value, when they are low in probability but potentially extreme in size.

We might imagine that countries are strategically connected to *H*ome and *F*oreign in some other way, unrelated to security. Perhaps there is something economically or culturally unique about *H*ome and *F*oreign that war would undermine. It could be that, for humanitarian reasons, these outside countries internalize some degree of the costs of war imposed on the populations of *H*ome and *F*oreign. Our results clarify that it is only in these types of settings that outside countries would be willing to make strategic loans that potentially violate the preventive war logic of this paper. In this setting, the only limit on the terms of loans that could prevent war would be the extent of the cultural or economic interest in avoiding war or the level of internalization of harm to other countries. We briefly consider the influence of strategic countries with outside money that are not connected with regard to security in the following section.

Finally, in an extended model with an endogenously determined risk-free rate of return and many states acting as potential borrowers and lenders, power shifts and war have the potential to transmit economic implications throughout the system. Power shifts that do not cause war but require borrowing to remain peaceful act as a positive demand shock on credit. On the other hand, full wars act as a negative supply shock as the warring states lose resources to loan on international markets. As mentioned before, this may lead to a contagion effect of conflict which causes r to rise endogenously, pushing other peaceful settlements out of reach. Short of war, the increase in r would also impact the distribution of the economic pie. Suppliers of capital would benefit from the improved terms, while demanders of capital would suffer from increased rates. Higher rates may still hurt lenders if they cause borrowers to default on their loans.

Strategic lending with outside money

There are many reasons why a country that is not connected to *H*ome or *F*oreign in a way that is related to security may nonetheless prefer one outcome to another. These preferences might be the result of a multitude of interests, including socio-cultural and economic interests. In this section, we consider a state that prefers a peaceful resolution when war between *H*ome and *F*oreign is imminent.

Suppose that a country A is outside of *H*ome and *F*oreign's security network and has a preference for peace. A simple case is that A is deeply humanitarian and incurs costs when anyone in the world is at war. Alternatively, we can suppose A has economic incentives that lead it to prefer peace between *H*ome and *F*oreign. For example, if country A heavily imports rubber and *H*ome has a comparative advantage in the production of rubber, A might expect *H*ome to channel more resources into security and less into production in the shadow of war. Even if A does not intend to import rubber directly from *H*ome, *H*ome's increased production will nonetheless lead to an increase in the global supply, which would reduce A's purchase price. Therefore, A would anticipate improved terms of trade from a peaceful resolution to the impending conflict between *H*ome and *F*oreign.

In any situation like this, A can advance a peaceful resolution by lending to Home when the lender's or borrower's constraint is violated. Avoiding war will require A to agree to a higher price on H's debt than the bond market would be willing to accept, so A will only lend if war is otherwise imminent and the gain from peace outweighs the loss from accepting a higher price. To avoid uninteresting cases, we assume country A has the financial ability to offer such a loan. A will never incur losses larger than necessary to facilitate peace and so, if it decides to offer a loan, it will leave the weakest of the violated constraints binding. When peace-generating strategic lending occurs, a preference for peace can be analogous to a preference for Home. For example, this can happen if Home prefers not to fight but the lender's constraint is violated. A's loan then makes peace possible and increases Home's expected payoff.

Country A must decide whether it is willing to take a lower bond repayment when the lender's or borrower's constraint is violated. The losses A is willing to take on the loan are limited by its gains from a peaceful outcome. Recall from Lemma 3 that H must transfer qB^* to F in order to avoid war. Since the price is a function of the probability of default and the risk-free rate,

$$q = \frac{1-\delta}{1+r}$$

if the borrower's or lender's constraint is violated, that means the only possible change to this relationship that A can make is to lend at a rate r^A that is less than the risk-free rate. Since the value of the transfer to F is set at qB^*, this higher price means that A will not see the return B^* in period 2, but some lower amount B^A where $B^A = (q/q^A)B^*$ or, equivalently,

$$B^A = \frac{1+r^A}{1+r}B^* \quad ^{22}$$

First, suppose the lender's constraint is not violated but the borrower's constraint is. Then, inequality (3) does not hold but inequality (2) does, implying that, while the bond market is willing to offer a loan, Home prefers the costs of war to the cost of the debt incurred by issuing a bond. Since the right-hand side of inequality (2) is decreasing in r, a lower rate r^A is required to satisfy H. To encourage Home to issue debt and avoid war, A needs to be willing to lend at price q^A, defined as a function of r^A, such that

$$\theta\rho = \rho + \frac{1-\beta}{\beta} - \frac{\kappa}{s}\left(\beta + \frac{1-\beta(1-r^A)}{\beta[1-\beta(1+r^A)]}\right) \tag{4}$$

holds. Satisyfing equation (4) requires

$$r^A = \frac{\beta\left[\rho\left(\frac{1}{1-\beta}\theta s - \kappa\right) + (1-\rho)\left(\frac{1}{1-\beta}s - \kappa\right)\right] - \frac{1}{1-\beta}s + \kappa}{\beta s\left[\frac{\beta\rho(\theta-1)}{1-\beta} - \left(1 + \frac{1+\beta}{s}\kappa\right)\right]} - 1,$$

from which we can recover q^A. Under this new price, while war can now be avoided, neither Home's nor Foreign's expected utility has changed.

Alternatively, suppose the borrower's constraint is not violated but the lender's constraint is. This time, inequality (2) does not hold but inequality (3) does. While Home prefers a

peaceful resolution and would like to issue debt, the bond market will not offer a loan in the amount required to placate *F*oreign, as the amount to be paid back is too great given the market price q. In order for A to lend, they need to offer a higher price q^A on a lower bond value B^A such that

$$\frac{s}{\rho}(1+r^A)\left[\frac{\beta\rho(\theta-1)}{1-\beta}-\left(1+\frac{1+\beta}{s}\kappa\right)\right]=\beta\left(\frac{1}{1-\beta}\theta s-\kappa\right) \tag{5}$$

holds. Satisfying equation (5) requires

$$r^A=\frac{\beta\rho\left(\frac{1}{1-\beta}\theta s-\kappa\right)}{\frac{s\beta\rho(\theta-1)}{1-\beta}-s\left(1+\frac{1+\beta}{s}\kappa\right)}-1,$$

from which we can recover q^A and B^A. As opposed to the first case, this new price both avoids war and improves *H*ome's expected utility. A preference for peace can be analogous to a preference for the rising state.

If both the borrower's and the lender's constraints are violated, the minimum of these two possible values for r^A is used to recover q^A and B^A. Let r^α equal the rate r^A from equation (4) when only the borrower's constraint is violated, r^A from equation (5) when only the lender's constraint is violated, or the minimum of these two when both are violated. Country A will be willing to offer the higher price

$$q^\alpha=\frac{1-\delta}{1+r^\alpha}$$

in order to receive the gains from peace so long as it is sufficiently likely that *H*ome can pay them back. Here, we assume country A's gain from peace is not larger than the total amount qB^*. Otherwise, A would not face a lending decision—they would always prefer to simply gift *H*ome the full amount qB^*. A is willing to purchase H's debt with the bond amount B^α at a price q^α (such that $q^\alpha B^\alpha=qB^*$) so long as the gain from peace is greater than or equal to the corresponding losses. By assumption, A could instead lend the amount qB^* on the market instead of to *H*ome. Lending the same amount on the market has an expected return equal to $(1+r)qB^*$, whereas the return from lending to *H*ome is $(1-\delta)B^\alpha$. Since

$$qB^*=\frac{1-\delta}{1+r^\alpha}B^\alpha,$$

the loss A incurs by facilitating a peaceful resolution is equal to

$$(1-\delta)\frac{r-r^\alpha}{1+r^\alpha}B^\alpha.$$

This result is stated in the following lemma.

Lemma 6. *When inequality (2) or (3) is satisfied, a peace-motivated country A will use outside money to facilitate peace if and only if A's gain from peace in period 2 is at least*

$$(1-\delta)\frac{r-r^\alpha}{1+r^\alpha}B^\alpha.$$

While this section explored the strategic use of outside money to facilitate peace, there are many other motivations that could lead to the use of outside money. Countries might consider lending due to preferences that are in varying degrees of alignment with those of *H*ome and *F*oreign. The logic underlying these extensions generates the anticipated forces for war and peace.

Empirical implications

The theory presented here generates a number of predictions on how the outbreak of war depends on various macroeconomic factors. A core prediction specific to our model is that the probability of war rises in the cost of capital when preventive motives exist. As the risk-free interest rate increases, the rising state will either prefer fighting due to the greater amount of borrowing required to prevent war or fail to secure financing due to inevitable default at prevailing rates. We also know from existing research that states often borrow to fund military expenditures for war directly (e.g. Slantchev, 2012; Zielinski, 2016). Therefore, we predict that, while war should be decreasing in interest rates, war should also be increasing in the interaction between interest rates and preventive motives. This section provides empirical support for this claim.

We use data from the Correlates of War (COW) project and the Bank of England's Three Centuries of Macroeconomic Data Project (Hills and Thomas, 2010). Our starting point is the framework developed in Lemke (2003) to investigate the prevalence and bellicosity of the preventative motive between states on a sample of dyads between 1816 and 1992. The sample contains all initiator–target dyads that have gone to war between 1816 and 1992 and a sample of the larger population of potential initiator–target politically relevant dyads that avoided war.[23] Lemke (2003) uses the COW data on national material capabilities to estimate trends in power shifts within strategically relevant dyads. This measure is then paired with controls for regime type, alliances, and other factors to test if the presence of a preventive motive for war is associated with war onset. We marry Lemke's data with a long-running time series of the real risk-free interest rate from the Bank of England. For the early period of our data, up to 1929, we use the yield on consol bonds minus inflation expectations.[24] After 1929, we use the standard UK bond rate less inflation expectations.[25]

In setting out to recover the impact of preventive motives on the likelihood of war, Lemke quantifies preventive motives by calculating a state's share of dyadic power over a 20 year period and regressing against time. Negative and statistically significant slope coefficients are considered a necessary and sufficient condition for a declining trend in the state's relative power and the existence of a preventive motive. This measurement process is executed across two different measures of power, one using COW's composite capabilities index and another using only the military components of the COW capabilities measure. Owing to concerns about the accuracy of originator identification, Lemke generates an additional "non-directed" preventive motives indicator such that a preventive motive exists when either dyad member is in relative decline to the other. As a result, the analysis provides us with four estimates of interest.[26]

We run four logistic regressions models with war onset as the dependent variable and where each preventive motives estimate (PM) mentioned above is interacted with the real risk-free rate (Rate). Additionally, we run two logistic regressions with directed dyads, both

Table 2. Logistic regressions on war.

	(1)	(2)	(3)	(4)	(5)	(6)
PM	45.384**	14.384	169.142***	69.620***	39.939*	7.594
	(18.671)	(10.173)	(46.298)	(16.851)	(21.118)	(11.188)
Rate	−0.294***	−0.299***	−0.295***	−0.325***	−0.309***	−0.314***
	(0.034)	(0.035)	(0.040)	(0.040)	(0.039)	(0.039)
PM×Rate	20.204***	13.628***	3.159	9.383**	18.439***	13.126***
	(5.389)	(3.270)	(10.387)	(4.348)	(5.954)	(3.543)
Constant	−5.375***	−5.377***	−5.688***	−5.644***	−6.097***	−6.114***
	(0.106)	(0.107)	(0.128)	(0.123)	(0.169)	(0.170)
Average marginal	0.251**	0.084	1.155***	0.483***	0.601*	0.120
effects PM	(0.112)	(0.066)	(0.383)	(0.085)	(0.318)	(0.171)
Rate	−0.002***	−0.002***	−0.002***	−0.002***	−0.005***	−0.005***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)
PM×Rate	0.112***	0.080***	0.022	0.065**	0.277***	0.208***
	(0.030)	(0.017)	(0.067)	(0.034)	(0.101)	(0.068)
Dyads	Directed	Directed	Non-directed	Non-directed	Directed	Directed
Power	Overall	Military	Overall	Military	Overall	Military
Lemke controls	No	No	No	No	Yes	Yes
Observations	495	494	495	494	495	494
Log likelihood	−294.297	−291.664	−286.273	−283.581	−248.556	−246.789
Akaike information criterion	596.594	591.327	580.546	575.162	515.112	511.577

Note: The preventive motive (PM) estimate that is operative in each model is described by the Dyads and Power rows. The Dyads row indicates whether the estimate was recovered using dyads where the initiator is in relative decline (Directed) or where either dyad member is in relative decline (Non-directed). The Power row indicates whether the estimate was recovered using COW's composite capabilities index (Overall) or only the military components of the COW capabilities measure (Military). The Lemke Controls row indicates if the model includes the control variables from Lemke (2003), which are indicator variables for whether the initiator is a democracy and whether dyads are allied, contiguous, or rivals. Here, the variable Rate is the real long rate provided by the Bank of England. Standard errors are in parentheses. Coefficients that are significantly different from zero are denoted by the following system: *10%, **5%, ***1% for the two-tailed test.

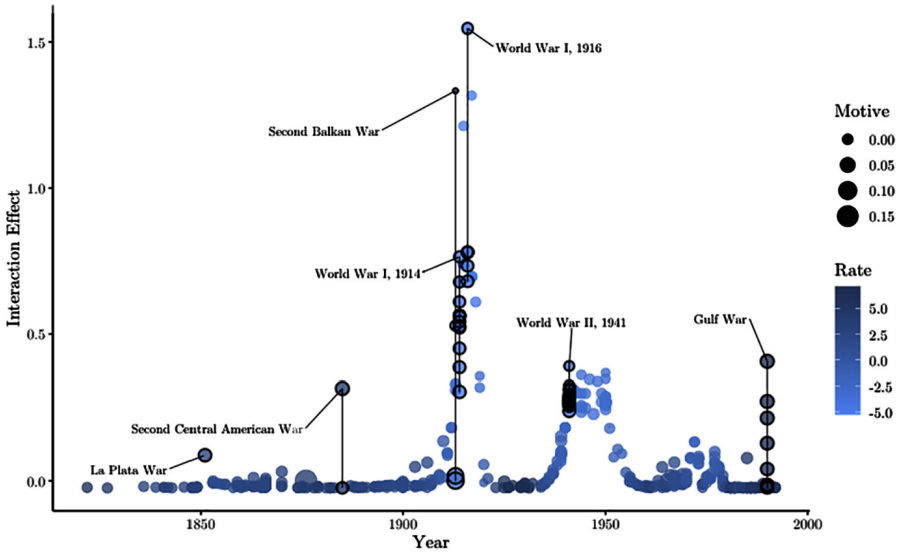

Figure 6. Interaction effects over time.

Note: Each point depicts an observation-specific interaction effect, with size according to the strength of the preventive motive and color reflecting the real rate. The interaction effects displayed here correspond to a model that uses Lemke's (2003) measure of preventive motive that was recovered with directed dyads and COW's composite capabilities index, the real rate provided by the Bank of England, and no additional controls.

measures of the preventive motive, and Lemke's controls. In Lemke's original article the controls are indicator variables for whether the initiator is a democracy and whether dyads are allies, contiguous, or rivals.

Table 2 presents our results. Unlike Lemke (2003), we find consistent support for his retrospective, trend-based assessment of the preventive motive on its own. We also find a consistent negative relationship between our measure of the risk-free rate and war, which aligns with the literature's expectation of the rate's effect on the direct costs of war. The interaction term for the preventive motive and the risk-free rate is consistently positive and almost always significant across all the model specifications. We find the same consistent positive relationship when we calculate the average marginal effect of the coefficients for our logit model. In every case, the effect of the interest rate is to increase the preventive motive effect. Or, put the other way, the preventive motive undermines the pacifying effect of costly debt.

We also calculate the observation-by-observation interaction effect, defined as the numerical value of the cross-partial derivative rather than simply the average marginal effect of the logit coefficient (see Ai and Norton, 2003). The results are consistent across specifications and, by breaking down the data to look at the interaction effects, we can see individual observations of the marginal effects track. For example, the top two deciles of the interaction effects calculated for individual observations contain the observations for World Wars I and II, while the bottom decile consists of examples like the US–Taiwan dyad in 1969 and UK–Mauritania in 1984.

To get a better substantive sense of the results, Figure 6 plots the interaction effect for dyads in our data over time. The x-axis is the year and the y-axis is the calculated value of the cross-partial derivative of the model with respect to the preventive motive and the rate at the values of the observation. The size of the dot represents the size of the measured preventive motive for the dyad in that year and the color is the observation's real risk-free rate.

The figure illustrates some facts about our results. First, while there is minimal variance in the preventive motive over time (the circle sizes are fairly stable), the effect of access to credit had its largest effect on war onset where expected. Somewhat muted by the size of the World War effects, we also find that, for A.J.P. Taylor's (1987: 166) examples of preventive wars (the War of Italian Unification, the Seven Weeks War, the Franco-Prussian War, and World War I), our model estimates an effect of the interest rate–preventive motive interaction.

Interestingly, we also see that the estimated effect was large at the beginning of the Cold War, but dropped precipitously once the Soviet Union became a nuclear power. Our estimates show that the effect was large between Iraq and some of the allies in the First Gulf War.[27]

Conclusion

In this paper, we built a model of commitment problems and sovereign lending. We have demonstrated that war resulting from commitment problems may still occur in this setting and that the potential for conflict is increasing in the expected size of a power shift. Our analysis also generates new empirical implications. First, we show that extreme but unlikely shifts are more dangerous than moderate but likely ones. Second, by ignoring the role financial markets might play in the bargaining theory of war, we both overestimate the likelihood the commitment problem is widely relevant and underestimate the resources available to support peaceful solutions, especially in the modern period. Third, exogenous increases in the real risk-free rate will increase the potential for conflict when commitment problems and the preventive motive are present.

In the final section, we show that an analysis of war onset since 1816 provides preliminary evidence that bond market conditions can affect the prospects of peace in times of a power transition. Building on Lemke's (2003) earlier work, we find a positive effect of the preventive motive on war, a negative direct effect of the real risk-free rate. When the preventive motive is present, however, increases in the real risk-free rate generate a countervailing force pushing states toward war.

The model we pursued here is highly simplified. A number of immediate extensions would provide further insight on the connections between international finance and conflict. Allowing for endogenous military spending may add an interesting dimension where potentially rising states avoid preventive war by actively constraining future military spending, building debt, and spending it on non-military social programs. Additionally, more general approaches to modeling power shifts, fighting, and sovereign borrowing may identify a number of more nuanced results.

Particularly relevant extensions would bring a greater level of sophistication to the economic side of the model. First, while our model sets the risk-free rate exogenously, this rate will, in reality, vary with the endogenous demand on capital. When states demand bonds for non-productive reasons, such as avoiding war, this demand shock for capital raises the risk-free rate for all other borrowers. This means less financing for productive economic activity

and higher prices on capital for any other states needing to borrow due to shifts in power—perhaps even causing a war that would have been avoided when the risk-free rate was lower. Second, persistent shifts in power arise naturally in this setting as the result of long-term economic growth. A full macroeconomic model of the relevant countries would provide a greater understanding of how power shifts caused by economic growth relate to credit access. Indeed, declining powers and potential financiers will both be deeply concerned about the duration of a positive economic shock, though in almost directly opposite ways. Third, building in consumption smoothing and other reputational concerns over access to credit markets would allow the model to endogenously determine the credibility of paying back loans.

There are few, if any, models connecting the bargaining model of war with the deep literature studying international macroeconomics and finance. Even from the simple model presented in this paper, a number of non-obvious empirical connections arise between macroeconomic indicators and the potential for conflict. Beyond the results presented here, this research hopes to contribute to both literatures by providing a framework on which to build more sophisticated models at their intersection.

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Supplemental Material

Supplemental material for this article is available online.

Notes

1. For instance, Baliga and Sjostrom (2013) comment that commitment problems as a cause of war arise when transfers are limited to current output. They note that transfers are limited in this manner when international bankers are unwilling to lend to states.
2. The expected size of a power shift is the probability that the shift occurs times the magnitude of the shift.
3. Several studies, including Coudert and Mignon (2013), demonstrate that the carry trade with South Korea can produce excess returns in normal economic times. Rare economic disasters have been put forth as an explanation for why excess returns in the carry trade persist (Farhi and Gabaix, 2016). Barro (2006) explicitly links rare economic disasters with the possibility of warfare.
4. In an empirical paper, Chapman and Reinhardt (2013) find that higher costs of foreign capital increase the likelihood of civil conflict.
5. See Kindleberger (1973) and Tooze (2006) for in-depth analyses of this case.

6. A particularly interesting connection between financial markets and war is the implication that significant conflict in one part of the world may raise the risk-free rate and serve as a contagion channel for war in other parts of the world.
7. There is a long tradition in international relations that studies preventive war going back to Thucydides's *The History of the Peloponnesian War*. The idea was further developed by Organski and Kugler (1981) and Levy (1987), before it was taken up in the context of crisis bargaining.
8. Like McDonald (2009), we are also making an argument about the way that capitalism can foster peace, but by a different mechanism.
9. Powell (2006: 192–194) analyzes a different context where the cost of maintaining the status quo leads to war.
10. For example, see Hassner's (2003) observations about the difficulty of negotiating over Jerusalem at Camp David in 2000.
11. Alternatively, the lender may be another state. In this case, the lender may be strategic, a possibility we consider below.
12. In this sense the financial resources are coming from outside the strategic interaction. We refer to this later as outside money. The case for inside money is considered in an extension.
13. We explore these relationships empirically below.
14. To focus on preventive war and commitment problems we only consider positive shocks to H's exogenous probability of victory. The exogenous change in probability is for clarity of presentation. One could micro-found this change by considering shocks to cost of effort in a contest or other factors that have a natural effect on the willingness to spend resources on military capabilities.
15. The proof of all lemmas can be found in the Supplemental Material.
16. It is possible money could be borrowed to increase a state's strength tomorrow rather than for a transfer today. If money borrowed by the rising state was used to make the power shift even bigger in the next period, then war would be more likely, especially with low interest rates. If, instead, it were the declining power who borrowed, the consequences would depend on a couple of factors. Paying the cost of borrowing to improve your payoffs in the future follows the same basic logic as paying the cost of war to lock in a better stream of benefits. Whether the choice to borrow turned out to be a worthwhile investment would depend on how much needs to be borrowed, what the possible return to investment might be with respect to the power shift, and whether the "fix" would be permanent, or just push the reversal in relative power off to some future date. It would be interesting to think about power shifts forestalled by borrowing in a world where leaders are short-lived relative to the state, but we leave that to future research. In any case, after all these considerations were made, we would then be back at the initial conditions of our model and we would then ask: is a preventive war on the offering and can access to debt smooth over the conflict?
17. Note that if $qB^* < 0$, then no borrowing is required.
18. The assumption of total default is based on the idea that the inability to pay back the bond makes future borrowing for any purpose impossible, so there is no point in paying back part of the debt. If instead we assumed that there was partial default, it would have the natural effect of making risky lending more attractive and risky borrowing more costly, but not fundamentally change any result.
19. Note that in traditional macro models, H would borrow in order to smooth consumption across periods. This incentive does not come into play in this model since we have assumed linear utility in consumption.
20. The proofs for Proposition 1–3 can be found in the Appendix. The proof for Proposition 4 can be found in the Supplemental Material.
21. Note that, all else equal, relaxing assumptions (1) or (2) just makes the result that preventive war exists in this setting easier to obtain.

22. Note that the value of r^A varies depending on whether indifference between peace and war leaves the lender's or the borrower's constraint binding.
23. This retrospective sampling technique is explained in King and Zeng (2001) and we use it here to make our results comparable to existing work.
24. A consolidated annuity is a coupon bond that pays a fixed percentage of the bond's face value every year and has no explicit maturity date, though the government can redeem them at any time. Like that of US long-term Treasuries, the consol rate is often used as a measure of the risk-free rate of return on global assets between the 18th and 20th centuries (Barro, 1987; Barsky and Summers, 1988; Shiller and Siegel, 1977).
25. Since there is some disagreement about how to measure inflation expectations, in the Supplemental Material we provide results just using the consol interest rate. This is clearly the risk-free rate but does not account for inflation. The results relating to the interaction of the preventive motive and the interest rate are robust—although the effect of the preventive motive alone is not.
26. A replication of Lemke (2003) is available in the Supplemental Material.
27. Another recent article by Bell and Johnson (2015) uses a different (prospective) measure for the preventive motive. We provide a specification with their measure in the Supplemental Appendix.

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Appendix

Proposition 1

Proof. The first statement holds since parameters exist where both (1) and either (2) or (3) is satisfied. For instance, fix a θ that satisfies (1). Then (3) holds when the bracketed value is positive (this is assured when κ is small and β is close to 1) and r is large.

For the second statement, there are three ways expected shift size can increase. Either θ increases, ρ increases, or both increase.

θ increases: Increasing θ directly increases the left-hand side (LHS) of (1), while the right-hand side (RHS) is unaffected, which directly increases the range of parameters that satisfy inequality (1). Increasing θ directly increases the LHS of (2) while the RHS is unaffected, which directly increases the range of parameters that satisfy inequality (2). Finally, increasing θ causes the LHS of (3) to increase faster than the RHS of (3), since taking the derivative with respect to θ of both sides gives

$$s(1+r)\frac{\beta}{1-\beta} > s\frac{\beta}{1-\beta},$$

since $r > 0$ by definition. Therefore, increasing θ increases the range of values that satisfy (3). For all three inequalities, increasing θ increases the potential for conflict.

ρ increases: Increasing ρ lowers the RHS of (1) without affecting the LHS, which increases the range of parameters that satisfy inequality (1).

For (2), subtract ρ from each side to get

$$(\theta-1)\rho > \frac{1-\beta}{\beta} - \frac{\kappa}{s}\left(\beta + \frac{1-\beta(1-r)}{\beta(1-\beta(1+r))}\right).$$

Since $\theta > 1$ by definition, the LHS is increasing in ρ while the RHS is unaffected. Hence, increasing ρ increases the range of parameters that satisfy (2).

For (3), the RHS is unaffected by ρ. Rearranging the LHS gives

$$s(1+r)\left[\frac{\beta(\theta-1)}{1-\beta} - \frac{1}{\rho}\left(1 + \frac{1+\beta}{s}\kappa\right)\right].$$

Since the second term in the brackets is negative and decreasing in ρ, the LHS is increasing in ρ. Hence (3) will be satisfied for a larger range of parameters.

Finally, if expected shifts increases owing to an increase in both θ and ρ, the above two cases demonstrate that the range of parameters satisfying (1), (2), and (3) will all increase.

Proposition 2

Proof. For two shifts of the same expected size, it is clear from the equation immediately above Lemma 3 that the amount borrowed to prevent war remains the same. Label the first shift (θ, ρ) and the second shift (θ', ρ') where $\rho' < \rho$ and $\theta' > \theta$, since the shifts have the same expected size. Assume that a positive amount must be borrowed to prevent war under (θ, ρ), and label the borrowed amount necessary to prevent war, qB^*. Similarly, let $q'B'$ be the amount necessary to prevent war under (θ', ρ').

Since the expected size of the shifts are the same, it must be that, if there is any risk of default at all,

$$qB^* = q'B'$$

$$\frac{\rho}{1+r}B^* = \frac{\rho'}{1+r}B'$$

$$B' = \frac{\rho}{\rho'}B^*.$$

That is, $B' > B^*$ since $\rho' < \rho$. Specifically, plugging in for B^* we get

$$B' = \frac{s}{\rho'}(1+r)\left[\frac{\beta\rho(\theta-1)}{1-\beta} - \left(1 + \frac{1+\beta}{s}\kappa\right)\right].$$

Note that it cannot be the case that (θ, ρ) leads to a no-default loan, but (θ', ρ') leads to a risky loan. The bond market offers a no-default loan for (θ, ρ) when

$$s(1+r)\left[\frac{\beta\rho(\theta-1)}{1-\beta} - \left(1 + \frac{1+\beta}{s}\kappa\right)\right] < \beta\left(\frac{1}{1-\beta}s - \kappa\right).$$

Since $(\theta-1)\rho s=(\theta'-1)\rho' s$, the market should also offer a no-default loan for (θ',ρ'). But if these values lead to a no-default loan, then inequality (3) cannot be satisfied, so, for this inequality in isolation, the potential for conflict is invariant in ρ'.

Inequality (3) is satisfied and war occurs under (θ',ρ') when

$$\frac{s}{\rho'}(1+r)\left[\frac{\beta\rho(\theta-1)}{1-\beta}-\left(1+\frac{1+\beta}{s}\kappa\right)\right]>\beta\left(\frac{1}{1-\beta}\theta' s-\kappa\right).$$

From the definition of shifts of the same expected size, we have

$$(\theta-1)\rho s=(\theta'-1)\rho' s$$

$$\theta'=\frac{\theta-1}{\rho'}\rho+1.$$

Plugging this in to the RHS gives

$$\frac{s}{\rho'}(1+r)\left[\frac{\beta\rho(\theta-1)}{1-\beta}-\left(1+\frac{1+\beta}{s}\kappa\right)\right]>\beta\left(\frac{1}{1-\beta}\left[\frac{\theta-1}{\rho'}\rho+1\right]s-\kappa\right).$$

Multiplying through by ρ' results in

$$s(1+r)\left[\frac{\beta\rho(\theta-1)}{1-\beta}-\left(1+\frac{1+\beta}{s}\kappa\right)\right]>\beta\left(\frac{1}{1-\beta}[\theta-1\rho]s+\rho'\left(\frac{s}{1-\beta}-\kappa\right)\right).$$

Thus, since $s/(1-\beta)>\kappa$, giving a positive war value in the first period, the RHS is increasing in ρ', whereas decreasing ρ' (shift becomes more extreme, lower probability) lowers the RHS. This makes (3) easier to satisfy.

For (1), we can manipulate the inequality to get

$$(\theta-1)\rho s>\frac{1-\beta}{\beta}(s+(1+\beta)\kappa).$$

The LHS is invariant by definition of equal expected shifts, and the RHS is invariant since it does not depend on θ or ρ. Similarly for (2), we can manipulate the inequality to get

$$(\theta-1)\rho s>\frac{1-\beta}{\beta}s-\kappa\left(\beta+\frac{1-\beta(1-r)}{\beta(1-\beta(1+r))}\right).$$

Once again, the LHS is invariant by definition of equal expected shifts, and the RHS is invariant since it does not depend on θ or ρ. Thus, only inequality (3) sees a change.

Proposition 3

Proof. r is not present in inequality (1), so this inequality is unaffected by changes to r.

For (2), r is not present in the LHS. Take the derivative of the RHS with respect to r:

$$-\frac{\kappa}{s}\left(\frac{\beta}{\beta(1-\beta(1+r))}-\frac{1-\beta(1-r)}{[\beta(1-\beta(1+r))]^2}(-\beta^2)\right)$$

$$-\frac{\kappa}{s}\left(\frac{\beta^2(1-\beta(1+r))}{[\beta(1-\beta(1+r))]^2}+\frac{\beta^2(1-\beta(1-r))}{[\beta(1-\beta(1+r))]^2}\right)$$

$$-\frac{\kappa}{s}\left(\frac{2-2\beta}{[1-\beta(1+r)]^2}\right)$$

The term in parentheses is positive, so increasing r is negative for the RHS. So, overall, increasing r decreases H's willingness to borrow.

For (3), r is not present in the RHS, and the derivative of the LHS with respect to r is

$$s\left[\frac{\beta(\theta-1)}{1-\beta}-\frac{1}{\rho}\left(1+\frac{1+\beta}{s}\kappa\right)\right],$$

which is positive so long as the bracketed amount is positive. Since this amount must be positive for (3) to be satisfied, increasing r increases the LHS and makes (3) easier to satisfy.