Bargaining, War, and Cooperation in the Long Run

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March 29, 2022

Abstract

Maintaining peace is costly. To understand what this implies about war outbreak and frequency, this article provides a dynamic crisis bargaining model where peace is costly and countries can take diplomatic action to compete over the bargaining surplus. Against conventional wisdom, repeated interactions of patient countries can destabilize peace. The likelihood of war relies on fundamentals of the international order, such as the persistence of war outcomes and peace agreements, as well as the severity of competition in diplomatic affairs. Even when countries prefer to cooperate, inadvertent wars are inevitable in the long run due to a coordination problem induced by costly peace. Diplomatic competition reduces the gains from peace and affects the probability of inadvertent war, but does not directly instigate attacks. The model offers new explanations for war, highlights the importance of institutional design in averting conflict, and sheds light on which international orders are most likely to fare well over time.

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Thanks to Peter Bils, Perry Carter, Zenobia Chan, Brendan Cooley, Germán Gieczewski, Matias Iaryczower, Gleason Judd, Gaétan Nandong, Kris Ramsay, Esther Robinson, Arantxa Rodriguez-Uribe, and Grace Zeng for their helpful comments and suggestions.

1 Introduction

Maintaining peace with adversaries requires time, money, and effort. Despite this, formal models of conflict typically depict cooperation as a costless byproduct of not fighting. The assumption that peace is free is not without loss of generality and affects our understanding of what causes war. This article provides a dynamic crisis bargaining model where peace is costly and countries can take diplomatic action to compete over the bargaining surplus. The findings contradict conventional wisdom from conflict studies and international relations theory generally.

Countries take the costs of peace seriously in contemporary global affairs. In 2011, then-Secretary of Defense Robert Gates warned NATO that the United States would not continue funding a disproportionate share of its costs—an issue that Donald Trump successfully campaigned on in the 2016 presidential election. Likewise, the United Kingdom withdrew from the European Union primarily due to public opinion that the cost to British sovereignty was too high, as membership would reduce their ability to regulate national immigration policy. Such developments have led many to question the fate of the prevailing liberal international order, with some scholars arguing that failure is inevitable (Mearsheimer 2019), while others remain optimistic for a rebound (Ikenberry 2018).[1] This paper does not take a stand on what lies ahead

[1]For more on the establishment of a so-called liberal hegemony, see Ikenberry (2001).

for the current international order, but instead provides a theory that informs how changes in international orders affect the likelihood of war.

The results of the model have direct implications on the outbreak and frequency of crises, which we can use to reflect on important questions facing international politics today. The model shows that patient countries may prefer to initiate conflict when war outcomes persist and settlements are transient. To understand what persistence of outcomes in the model represents, consider the Dayton Accords that resolved the Bosnian War in the mid 1990s. This is an example of transient war outcomes and highly persistent peace agreements, as Serbia understood defection and subsequent expropriation of territory would result in a high likelihood of conflict with the U.S. and NATO. This contrasts sharply with short-lived armistice agreements in civil conflicts, where more powerful parties capable of affecting change in the event of defection and conquest are unwilling to. In this sense, the persistence parameters of the model could be understood as a reduced form for a larger international setting to which the countries of the game are subject.

The model provides a framework through which we can improve our ability to explain and predict international crises based on observable changes to the international order. Take, for example, the 2022 Russian invasion of Ukraine. What prompted Vladimir Putin to launch an invasion twenty years after the dissolution of the Soviet Union? While there are reasonable explanations that already exist (e.g., Russia could be engaging in a screening process or, as John Mearsheimer contends, there could be a commitment problem resulting from NATO expansion), this article provides two new possibilities. First, shifts in the fundamentals of the international order may instigate conflict. As the model demonstrates, increasing the persistence of war outcomes or the transience of peace agreements can induce a sufficiently patient country to choose to fight. Then, for example, weakening global institutions and corresponding alliances could foment Russia's aggression. Second, war may result from a failure to coordinate a peaceful settlement. If the costs of participating in international organizations have risen, this will increase the likelihood countries forgo attempts to resolve their disputes.

Further, the model may help anticipate conflict that has yet to occur. Consider the possibility of war between the U.S. and China as an example. Scholars and policy analysts continue to debate whether China's growing power poses a threat to the liberal international order and, more specifically, whether China aspires to displace the United States as a global hegemon (Doshi 2021). A particular source of tension between the two is Taiwan's sovereignty. In 1975, during a conversation with then-Secretary of State Henry Kissinger, Mao Zedong remarked on China's hope to one day claim Taiwan—"a hundred years hence we will want it, and we are going to fight for it." Seen through the lens of this model, Mao's statement may indicate that China is indeed a patient country and a forceful attempt to annex Taiwan can be triggered by shifts in the international order that affect the persistence of outcomes.

1.1 Costly Peace

Few scholars of international cooperation would argue that peace is entirely free—nearly all work that studies international cooperation directly engages with the fact that doing so is costly. A driving motivation to establish international organizations is their ability to potentially reduce transaction costs (Keohane 1984). While it is possible or even likely that institutions—be it formal organizations, norms, or otherwise—can reduce these costs of peace, it is also worth considering the possibility that current institutions occasionally exacerbate this problem. Costs to reach agreements can lead to inefficiencies and hence an increased unwillingness to cooperate under suboptimal peace outcomes (Moravcsik 1999). According to Lake (1999), the transaction costs of international institutions determine the structure of security relations, varying from anarchic alliances to hierarchical ones—e.g., NATO and the Warsaw Pact, respectively.

Incorporating costly peace in the bargaining framework parallels with some of the recent literature on international cooperation and contributes to a larger conversation on its stability. For example, consider De Vries, Hobolt, and Walter's (2021) study on the effect of domestic public discontent and the "politicization" of international cooperation on its prospects. They argue that politicization can either destabilize cooperation by amplifying the demands of the opposition or stabilize cooperation by bolstering legitimacy. In the framework I lay out, these simplify nicely to heightening and reducing costs of peace, respectively. In another recent paper, Pratt (2020) shows that states create new institutions that better reflect their strength after experiencing favorable power shifts, leading to "institutional proliferation." Existing institutions therefore may impose additional costs on states seeking peaceful settlements if constructing new, alternative institutions in their presence is more difficult. Brutger (2021) argues that proposal power allows some leaders to avoid domestic audience or reputational costs that may otherwise undermine the prospect of international cooperation. Davis and Pratt (2021) show that geopolitical interests affect which countries join and remain in international organizations. The model I present here is consistent with these ideas, reconciling them with the canonical bargaining model of war and shedding light on some of the underlying mechanisms.

Another feature of institutional cooperation that has been less explored is that countries who incur the cost of peace also tend to reap its greatest rewards.[2] For example, voting with powerful countries on security issues has been shown to be associated with better outcomes in international organizations, such as a more favorable distribution of financial resources (Thacker 1999; Stone 2002; Stone, Slantchev, and London 2008; Stone 2011; Woo and Murdie 2017). This model accounts for this dimension of institutional cooperation by introducing competitive diplomatic spending. The country that spends the most on diplomatic endeavors will have an advantage in bargaining—specifically, it will be more likely that they are recognized with proposal power. As the article will show, the incentive to compete for proposal power will not directly lead countries to launch wars; however, competitive spending nonetheless affects both the welfare of countries and the frequency of inadvertent wars brought about by an underlying coordination problem.

While this article focuses attention on an institutional environment, the model can also be applied to more general settings in which countries incur costs from cooperating. Consider, for example, the costs imposed by a domestic audience that prefers to fight with an adversary (Fearon 1994; Tomz 2007; Snyder and Borghard 2011). Like diplomatic spending in the model, the costs of cooperating with such an adversary will be increasing in a country's effort to achieve peace, although such effort may result in preferable settlements for the country. Consistent with the model's results, Crisman-Cox and Gibilisco (2018) show that the likelihood of war is increasing

[2]See Vreeland (2019) for a review on how some countries are receive favorable treatment by international organizations.

in a country's audience costs.

1.2 Dynamics of Crisis Bargaining

This paper presents a dynamic crisis bargaining model and is therefore related to many models of conflict that precede it. Since Schelling (1960) argued that conflict was essentially a bargaining situation, a long tradition of scholarship has used the bargaining framework to gain insight into the nature of international war and cooperation (Blainey 1973; Iklé 1991). Fearon (1995) argues that rival countries should never fight costly wars in the absence of private information and commitment problems, as peaceful settlements necessarily exist. Powell (2006) identifies a country's incentive to renege on commitments as an important source of international conflict. Many scholars have subsequently extended the bargaining model to dive into the mechanisms underlying problems that bring about conflict, most notably problems pertaining to information asymmetries and commitment dynamics.[3] For example, recent literature has been active in exploring international bargaining under different types of uncertainty (Bils and Spaniel 2017; Spaniel and Bils 2018; Smith and Spaniel 2019) as well as war-inducing commitment problems in the context of international alliances, hassling, and civil war (Schram 2021; Benson and Smith 2021; Carey, Bell, Ritter, and Wolford 2022).

Two models that are especially relevant and worth mention are Powell (1993) and Fearon (1998). Powell (1993) studies the "guns versus butter" trade-off countries face. Each country has finite resources to allocate either to domestic ends, from which it derives direct utility, or to military means, from which it secures greater strength. As a result of this resource allocation problem, Powell discovers that a long shadow of the future encourages greater military allocations, which in turn makes war more likely as countries may prefer to fight in order to destroy their opponent and reallocate from military to domestic ends. The gain from future domestic ends as a result of victory in war may be large compared to the peaceful alternative. This finding is largely consistent with the results of this paper—it is patient countries, not the impatient ones, that may hold a rational preference for war over peace.

[3]Ramsay (2017) offers a review on information and war and Powell (2012) discusses commitment problems and war as a result of shifting power.

Powell's model showcases that the need to maintain strength for survival and hence expend resources building arms can itself promote war. The mechanism relies on a commitment problem—if we could do away with the threat of war, countries would no longer feel the need to arm heavily and could reallocate more resources to domestic ends without regard for their opponent's strength. Therefore, patient states may be driven to war due to their inability to commit to spending less on military arms. In this paper, however, the source of a country's failure to cooperate may be the result of the underlying fabric of the international order. When war outcomes are persistent and peace agreements are transient due to the international environment and not limitations in the countries' strategic behavior, there may be no commitment power we could endow countries with that would successfully avert war.

Fearon (1998) promotes a "bargaining first, then enforcement" approach when it comes to studying international cooperation. In the paper, Fearon develops a two-stage model where the the first stage features a war of attrition and the second stage features a Prisoners' Dilemma meant to represent an enforcement stage. The results indicate that the conventional cooperation theory reasoning about the shadow of the future is limited—the effect of the shadow of the future on international cooperation is ambiguous. In particular, a long shadow of the future can make it easier to enforce agreements but more difficult to arrive at agreements in the first place.

Unlike Fearon's model—where countries may hold out because they know agreements, once reached, will be enforced for a long time—in this paper, it is possible for peaceful settlements not to exist at all. Further, neither Powell's nor Fearon's model features the possibility of an inadvertent war. In the model herein, costly peace creates an incentive to free-ride on the necessary diplomatic expenses, which creates a coordination problem. Moreover, this paper studies competitive diplomacy, which is absent from previous crisis bargaining work and, in the context of this paper, is consequential for both welfare and the likelihood of inadvertent war.

Although the cooperation side of the interaction has not received adequate attention in the crisis bargaining literature, this article is not the first to address costly peace and its implications for war. Coe (2011) is especially noteworthy in that it not only argues for costly peace as a third rational explanation for war, but also analyzes several applied cases where the costs of peace led to war.[4] Coe's work focuses mainly on costly peace as an explanation for war when the costs of peace exceed the costs of war. Furthermore, Coe focuses his attention on non-institutional costs of peace—arming, imposition, and predation—which cannot improve the peacetime welfare of the country incurring the expense. This article, on the other hand, allows for costs of war that are larger than the costs of peace and studies competitive diplomatic spending in which favorable settlements are increasing in a country's expense.

Note that inadvertent war in this model is the result of a coordination problem and not a hold-up problem. Countries will simultaneously decide an amount to spend on diplomacy and, if aggregate spending is less than this threshold, cooperation cannot be maintained. Each country risks war in equilibrium by spending low amounts with nonzero probability. On the other hand, if aggregate spending exceeds the threshold, the country that incurred a larger diplomatic expense will receive a greater likelihood of being recognized as proposer in an ultimatum game. Fortunately, Fey and Kenkel (2021) show that an ultimatum game can be used to represent crisis bargaining with little loss of generality.

2 Model

Consider a game with two countries $i = 1, 2$ that have opposing preferences over the division of a unit interval. Each country's utility is continuous, increasing, and linear in their share of the pie each period.[5] There is a common discount factor $\delta \in (0, 1)$ over an infinite horizon, $t = 1, 2, \ldots$, where each period can be characterized by a pair of state variables $(b(t), p(t)) \in \{0, 1\} \times P$ with $P \subset [0, 1]$. The former denotes the bargaining status of the game, with $b(t) = 0$ if the countries are in a settlement and $b(t) = 1$ if the countries are bargaining. The latter provides the relative strength of country 1 in period t, where P is a finite set of possibilities.

Settlement is an inactive state in which each country simply consumes their share of the pie from the previous period. On the other hand, bargaining in period t

[4] The cases include the Iraq War, civil conflicts in Iraq after the Gulf War, and the American Revolution.

[5] Specifically, let $u_i(x)$ denote country i's flow payoff for a share $x \in [0, 1]$ of the pie such that $u_1(x) = x$ and $u_2(x) = 1 - x$.

permits each country a choice between cooperating with their opponent, $r_i(t) = 0$, or initiating war, $r_i(t) = 1$. If a country cooperates, they simultaneously choose an amount $s_i(t) \geq 0$ to spend in competition over proposal power. This amount can be likened to costly effort exerted in diplomatic relations, which may include embassy expenditures, ambassador salaries, etc., as well as financial contributions to an international organization that has credible domain over the conflict at hand. Exerting effort on cooperative endeavors provides countries with greater leverage in the bargaining game.

Cooperation is not necessarily free. In addition to the endogenous costs of cooperation, the model allows for an exogenous, context-dependent cost that serves as a required minimum on aggregate spending for peace, denoted by $k \geq 0$. An example of such a diplomatic spending threshold is the lower bound on necessary operating costs for an institution that enforces and monitors a contract between countries.[6] Then, if $s_1(t) + s_2(t) < k$ for some period t, peace cannot be maintained and the countries fall into war. Alternatively, cooperation is possible when $s_1(t) + s_2(t) \geq k$ and, given $s_i(t) > s_{-i}(t)$, country i is recognized as proposer in period t with probability $\pi > 1/2$.[7] The proposer makes a take-it-or-leave-it offer $x(t) \in [0, 1]$, with acceptance leading to peaceful settlement and rejection leading to war. An accepted division $x \in [0, 1]$ results in a periodic payoff of $u_1(x) = x$ for country 1 and $u_2(x) = 1 - x$ for country 2, less the costs spent on diplomacy.

War occurs in period t if and only if $b(t) = 1$ and either (i) at least one country chooses to attack, (ii) an offer is rejected, or (iii) aggregate spending is less than the exogenous spending threshold. Each country incurs a cost of war $c_i > 0$ and country 1 wins with probability given by state variable $p(t)$. The victorious country receives the entire pie for that period and subsequent periods until the game returns to bargaining.

The game begins in a bargaining stage by default so that $b(1) = 1$. At the end of any period $t \geq 1$, the state variable $b(t)$ transitions to $b(t+1) = 0$ with probability $\theta \in$

[6] While the aggregate spending threshold k is fixed in the model, allowing it to vary over time would not change the substantive results. The conclusions reached will be compatible with the idea that investing in cooperation today may ease future cooperation, i.e. $k(t) \geq k(t + 1) > 0$ for all $t \geq 1$.

[7] If $s_1(t) = s_2(t) > 0$, then proposal power is awarded to country 1 with probability 1/2. This, however, is a measure zero event in equilibrium.

$(0,1)$ when the current division is a war outcome and with probability $\lambda \in (0,1)$ when the division is the result of a peaceful settlement. These represent the persistence of outcomes settled by war and those settled by diplomacy, respectively. The state variable $p(t)$ transitions according to Markov transition function $q(p(t+1)|p(t), a(t))$ where $a(t) = ((r_i(t), s_i(t), x_i(t), y_i(t)))_i$.

2.1 Timing

The timing of game is as follows.[8]

1. Nature randomly draws initial relative strength $p \in P$ and sets $b = 1$.

2. If $b = 0$, the previous period's distribution of the pie persists. If $b = 1$, each country chooses to cooperate or fight, $r_i = 0, 1$.

2.a. If both cooperate, they choose $s_i \geq 0$. If $s_1 + s_2 \geq k$, a proposer is recognized according to π and the realized s_1 and s_2. The proposer i makes a take-it-or-leave-it offer $x_i \in [0,1]$, which country $-i$ rejects or accepts $y_{-i} = 0, 1$.

2.b. War occurs if a country attacks, an offer is rejected, or $s_1 + s_2 < k$. Costs of war $c_i > 0$ are incurred and country 1 wins with probability p.

3. At the end of any period $t \geq 1$, the state (b, p) transitions to (b', p') according to Markov transition function $q(p'|p, a)$ as well as probabilities θ and λ.

4. Period $t + 1$ proceeds from step 2 with state $(b = b', p = p')$.

2.2 Equilibria

Country i's payoff is a function of both countries' actions and the state variables and can be denoted by $\Sigma_i(b, p, a)$, which is simply a discounted sum of linear gains in a periodic share of the pie and linear losses in costs of war and diplomatic spending when they are incurred. A strategy for i is a function $\sigma_i : P \to \{0,1\}^2 \times [0,1] \times \mathbb{R}_+$, where $\sigma_i(\cdot)$ denotes behavior for country i when $b = 1$ for all $p \in P$, including their

[8] All notation that identifies a specific period t is suppressed when doing so does not create confusion.

war decision, diplomatic spending, bargaining offers, and bargaining responses. A country i's equilibrium strategy is given by

$$\sigma_i^*(p) = \arg\max_a \Sigma_i(b, p, a). \tag{1}$$

When $b = 0$, the countries take no action and simply consume their share of the pie according to the previous period's division. This allows us to focus on states in which $b = 1$. Denote country i's ex ante value function $V_i(p)$, which satisfies

$$V_i(p) = r_{-i}^*(p)(W_i(p) - c_i) + (1 - r_{-i}^*(p))\Big[r_i^*(p)(W_i(p) - c_i) + (1 - r_i^*(p))U_i(p)\Big], \tag{2}$$

where, for all states $p \in P$, $W_i(p)$ is country i's expected value from going to war, $U_i(p)$ is country i's expected value from cooperating, and $r_i^*(p)$ is country i's equilibrium war decision.[9]

If war occurs, each country can expect to receive the entire pie for that period with their probability of victory. In every period after war that the war settlement persists, the winning country can expect to keep receiving the entire pie while the losing country keeps receiving nothing with probability θ. The countries return to the bargaining game under a new state p' with probability $1-\theta$. Letting $p = p_1 = 1 - p_2$ without loss of generality, country i's expected value of war in state p can be expressed as $W_i(p) - c_i$, where

$$W_i(p) = p_i + \frac{\delta}{1 - \delta\theta}\left[\theta p_i + (1 - \theta)\sum_{p' \in P} V_i(p')q(p'|p, a)\right]. \tag{3}$$

Denote by $x_m(p)$ the expected peace settlement in state p. As a result of symmetric spending strategies, countries will earn proposal power half of the time they reach a peaceful settlement and hence $x_m(p)$ will be the Nash bargaining solution of the game. In every period after cooperation that the peaceful settlement persists, country 1 can expect to keep receiving $x_m(p)$ and country 2 can expect to keep receiving $1 - x_m(p)$ with probability λ. The countries return to the bargaining game under a new state p' with probability $1 - \lambda$. Then, we can also write country i's expected value of

[9] In abuse of notation, I regularly denote and refer to state $(1, p)$ as state p when $b = 1$ is implied.

cooperating in state p as

$$U_i(p) = u_i(x_m(p)) - \mathbb{E}[s_i^*(p)] + \frac{\delta}{1-\delta\lambda}\left[\lambda u_i(x_m(p)) + (1-\lambda)\sum_{p'\in P} V_i(p')q(p'|p,a)\right] \quad (4)$$

where $u_i(x_m(p))$ is country i's expected settlement given cooperation in p and $\mathbb{E}[s_i^*(p)]$ denotes expected diplomatic spending in state p according to i's equilibrium spending strategy.[10]

Throughout the paper, $U_i(p)$ refers to i's expected value for cooperating prior to allocation of proposal power. I will refer to the expected value of accepting an outstanding offer x as $U_i(x;p)$—therefore i's expected value of cooperating given $-i$ has won proposal power can be written $U_i(x^*_{-i}(p);p)$ for equilibrium offer $x^*_{-i}(p)$.[11]

Definition 1. *A* ***nearly symmetric stationary MPE*** *is a pair* $(\sigma_1(\cdot), \sigma_2(\cdot))$ *such that equations (1)-(4) hold for all* $p \in P$.

My solution concept is nearly symmetric stationary Markov perfect equilibrium (MPE). As in any stationary MPE, this solution concept satisfies the Markov property that strategies rely only on a payoff-relevant state. The term *nearly symmetric* emphasizes that countries act symmetrically in equilibrium with the exception that their behavior will necessarily rely on relative strength and potentially unequal costs of war.

The MPE solution concept is employed instead of subgame perfect Nash equilibrium (SPE) for two primary reasons. First, MPE focuses attention on characteristics of the underlying institution while avoiding equilibria that are facilitated through history-dependent strategies, which are outside of this study's scope, such as strategies that rely on reputation to facilitate international cooperation. Second, while reputation may be an interesting feature to understand in this context, SPE in this

[10]The expectation $u_i(x_m(p))$ is equal to a weighted average of i's share of the pie under equilibrium offers from each country. In equilibrium with symmetric recognition, the weights are necessarily 1/2 and hence $u_i(x_m(p))$ is the value i derives from a settlement at the Nash bargaining solution. Note the subtle difference that $u_i(x_m(p))$ is the value of the settlement to either country i, which is different from $x_i(p)$, the share of the pie retained by country 1 under an offer from country i. Specifically, $u_1(x_m(p)) = 2^{-1}\sum_i x_i^*(p)$ and $u_2(x_m(p)) = 2^{-1}\sum_i(1 - x_i^*(p))$.

[11]Note that $U_i(x_m(p);p) = U_i(p) + \mathbb{E}[s_i^*(p)]$ because the realized s_i becomes a sunk cost at the time of the proposal. For this reason, acceptance decisions will rely on $U_i(x;p)$ instead of $U_i(p)$.

model will allow for many unreasonable equilibria. Consider, for example, an SPE of this game where a country initiates war in some periods and cooperates in others, with off-path behavior such that deviations to cooperate result in punishment by other countries to fight wars forever thereafter. Such equilibria are not informative about international interactions. Limiting the analysis to MPE as defined above resolves these issues.

2.2.1 Offers and the Bargaining Range

To find equilibrium offers $x_i^*(p)$ for each country i in any state p, it is necessary to find the offer that makes each country indifferent between going to war and cooperating. By setting $U_1(\underline{x}(p); p) = W_1(p) - c_1$ and $U_2(1 - \overline{x}(p); p) = W_2(p) - c_2$, we can solve for values that make countries 1 and 2 indifferent in state p, respectively. These indifference conditions yield implicit solutions for the thresholds,

$$\underline{x}(p) = (1 - \delta\lambda)(W_1(p) - c_1) - \delta(1 - \lambda) \sum_{p' \in P} V_1(p')q(p'|p, a) \tag{5}$$

and

$$\overline{x}(p) = 1 - (1 - \delta\lambda)(W_2(p) - c_2) + \delta(1 - \lambda) \sum_{p' \in P} V_2(p')q(p'|p, a). \tag{6}$$

The relationship between these indifference settlements and the indifference settlements the standard static bargaining model of war is apparent. When $\delta = 0$, it is clear to see that $\underline{x}(p)$ becomes $p - c_1$ and $\overline{x}(p)$ becomes $p + c_2$, which is equivalent to the canonical characterization of the static bargaining range as in Fearon (1995).

Note that $\underline{x}(p)$ and $\overline{x}(p)$ are not constrained to feasible values in the unit interval. The peaceful division that makes a country indifferent between cooperation and war may therefore not be a valid offer. In these cases, the proposing country offers the best feasible offer in the bargaining range. Using these values, we can characterize the bargaining range in state p as the set $X(p) = [\underline{x}(p), \overline{x}(p)] \cap [0, 1]$ given $\overline{x}(p) \geq \underline{x}(p)$ and $X(p) = \emptyset$ otherwise.

The bargaining range will thus be nonempty when $\min\{1, \overline{x}(p)\} \geq \max\{0, \underline{x}(p)\}$. Exceptions when $\underline{x}(p) > 1$ or $0 > \overline{x}(p)$ result in a particular type of conflict known as preventive war. The source of the problem is a commitment problem—the division that one country would have to give the other to avert war is larger than the entire pie and there is no way to credibly promise future divisions. This may happen when a country's expected relative strength in a period after peace is small relative to that after war, e.g. if $\sum p' q(p'|p, a)$ is much smaller given $r_1 = 0$ than given $r_1 = 1$.

Unlike previous models, the bargaining range can also be empty without commitment problems. To see this, consider when $1 > \underline{x}(p) > \overline{x}(p) > 0$.[12] Then, conflict arises not because one state cannot offer an adequate division, but because they prefer not to settle. A trivial way this occurs in a static game is if the exogenous costs of peace are larger than the costs of war, which I assume away. The model herein, however, reveals that this type of problem may occur under more realistic conditions in which the costs of peace are much less than the costs of war. Therefore, dynamic considerations that lead to the depletion of the bargaining surplus are worth more serious investigation.

Regardless of cause, at least one country will prefer war in any period with an empty bargaining range. If the bargaining range is nonempty, each country will propose to allocate themselves the largest feasible division of the pie that their opponent is willing to accept. Then, we can define the equilibrium offers as

$$x_1^*(p) = \max\{0, \min\{1, \overline{x}(p)\}\} \tag{7}$$

and

$$x_2^*(p) = \min\{1, \max\{0, \underline{x}(p)\}\}. \tag{8}$$

Note that if $\overline{x}(p) < 0$ or $\underline{x}(p) > 1$, a country may not want to agree to their offer as defined by $x_i^*(p)$. For this reason, at least one country must prefer to initiate war when $X(p) = \emptyset$ even though they are indifferent between war and rejecting an offer.

Though the model can accommodate analysis of commitment problems, this is outside of the focus of this paper. Instead, the analysis here focuses on failures to cooperate brought about by dynamic features of international interactions that do not originate with previously studied rational causes of war. Thus, I proceed by making the following assumption.

[12] This is a sufficient but not necessary condition.

Assumption 1 (No commitment problems)**.** *For all* $p \in P$, $1 \geq \underline{x}(p)$ *or* $\overline{x}(p) \geq 0$.

This allows us to simplify our characterization of the bargaining range so that $X(p) = [x_2^*(p), x_1^*(p)]$ given $x_1^*(p) \geq x_2^*(p)$ and $X(p) = \emptyset$ otherwise.

2.2.2 Diplomatic Spending

With equilibrium offers defined, we can find equilibrium diplomatic spending. Spending is costly but provides each country with a chance to win the bargaining surplus with proposal power. Note that the value of earning proposal power is necessarily the same for each country conditional on the state p. In particular, the value of proposal power in any period is the length of the bargaining range in state p, which can be denoted by $B(p) = x_1^*(p) - x_2^*(p)$ for $x_1^*(p) > x_2^*(p)$ and zero otherwise. Given recognition probability $\pi > 1/2$ for contributing more than your opponent, the expected gain from being the highest contributor is $(2\pi - 1)B(p)$ in this period as well as each subsequent period with probability λ.

There is no equilibrium in pure strategies.[13] Instead, consider equilibrium spending such that each country mixes according to a cumulative distribution function (c.d.f.) denoted by $F(\cdot)$. When one country only spends nonzero amounts greater than or equal to k, the other country has no reason to spend nonzero amounts less than k either, as there is necessarily a profitable deviation to spending zero. Then, I look for an equilibrium spending strategy $F^*(\cdot)$ with $F^*(s) = F^*(0)$ for all $s \in [0, k)$. A strategy with nonzero spending less than k will be considered subsequently.

Each country i that contributes an amount $s \geq k$ has expected net utility

$$W_i(p) - c_i + \frac{\pi B(p)}{1 - \delta\lambda} F^*(s;p) + \frac{(1-\pi)B(p)}{1-\delta\lambda}(1 - F^*(s;p)) - s, \tag{9}$$

while spending zero yields an expected value of

$$W_i(p) - c_i + \frac{(1-\pi)B(p)}{1-\delta\lambda}(1 - F^*(k;p)). \tag{10}$$

Using the indifference condition, each country's equilibrium spending strategy $F_i^*(s;p)$

[13] This is standard for a common value all-pay contest. See the Appendix for a proof of the equilibrium spending strategy.

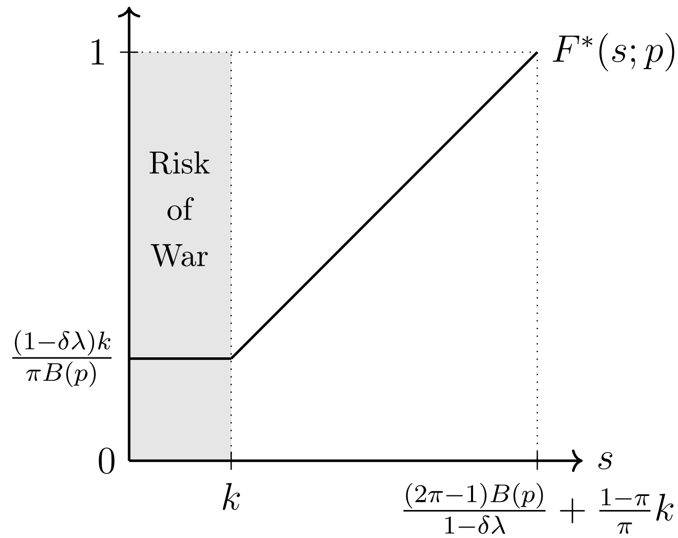

Figure 1. Equilibrium diplomatic spending according to c.d.f. $F^*(\cdot)$ with $\pi \approx 1$. Each country spends zero with probability $(1-\delta\lambda)k/(\pi B(p))$ and spends an amount greater than k with complementary probability. As $\pi \to 1/2$ from the right, countries lose the incentive to compete over the bargaining surplus and $F^*(\cdot)$ gets arbitrarily close to mixing between zero and k as in a normal coordination game. As $\pi \to 1$ from the left, countries spend more in competition over the bargaining surplus.

is well-defined for all $p \in P$. The result is illustrated in Figure 1 and stated formally in the following proposition.

Proposition 1 (MPE I)**.** *There is a nearly symmetric stationary MPE where, for all $p \in P$, each country $i = 1, 2$ plays $\sigma_i^*(p) = (r_i^*(p), s_i^*(p), x_i^*(p), y_i^*(x;p))$ defined as follows.*

(i) Initiate war, $r_i^*(p) = 1$, if and only if $W_i(p) - c_i > U_i(p)$.

(ii) Spend a random draw, $s_i^*(p)$, according to the distribution

$$F^*(s;p) = \begin{cases} 0 & \text{for } s < 0 \\ \frac{(1-\delta\lambda)k}{\pi B(p)} & \text{for } s \in [0, k) \\ \frac{(1-\delta\lambda)(s-\frac{1-\pi}{\pi}k)}{(2\pi-1)B(p)} & \text{for } s \in [k, \frac{(2\pi-1)B(p)}{1-\delta\lambda} + \frac{1-\pi}{\pi}k] \\ 1 & \text{for } s > \frac{(2\pi-1)B(p)}{1-\delta\lambda} + \frac{1-\pi}{\pi}k \end{cases}$$

if $U_i(p) \geq W_i(p) - c_i$, or else $F^*(s;p) = 1$ for all $s \geq 0$.

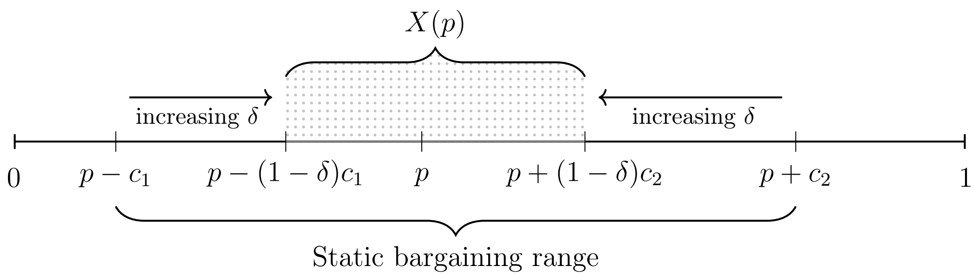

Figure 2. Patience can erode the canonical bargaining range when war outcomes are persistent and peace settlements are transient. Here is an illustrative example with $\theta \approx 1$ nand $\lambda \approx 0$. Assuming expected diplomatic spending is arbitrarily small, $X(p)$ will be arbitrarily close to $[p - (1-\delta)c_1, p + (1-\delta)c_2]$ as θ gets larger and λ gets smaller. The set of feasible settlements reduces to the singleton $\{p\}$ in the limit as $\delta \to 1$ from the left. Given an exogenous cost to cooperation $k > 0$, peaceful settlements will not exist with sufficiently high δ.

(iii) Offer $x_i^*(p)$ as given by equations (7)-(8) when recognized as proposer, or else accept an offer x, $y_i^*(x;p) = 1$, if and only if $U_i(x;p) \geq W_i(p) - c_i$.

War is sure to occur when the bargaining range is empty, whereas countries will opt for cooperation when the bargaining range is nonempty. This is analogous to standard bargaining models of war—preferable outcomes under peace can be facilitated over costly war due to complete and perfect information. While commitment problems are the sole source of nonempty bargaining ranges in standard models, the dynamic considerations of this model allow for other explanations. Figure 2 illustrates this with a simple example where $\theta \approx 1$ and $\lambda \approx 0$—that is, war is an absorbing state where the victor gets to keep the entire pie forever and peace agreements are short-lived. The dynamic bargaining range $X(p)$ is a proper subset of the static bargaining range and decreases in size as patience increases. When δ and θ are close to 1 and λ is close to 0, $X(p)$ is approximately equal to the singleton $\{p\}$. Then, it is easy to see how any arbitrarily small cost to cooperation $k > 0$ will instigate war.

The main difference in equilibrium behavior of this model and that of standard bargaining models is that countries react to costly peace. Peace is costly in two important ways: (i) the diplomatic spending threshold creates an inefficiency of cooperation akin to (albeit smaller than) the standard inefficiency of war and (ii) countries can now improve their settlement in peace through diplomatic spending. Each country has incentive to continue spending in competition for proposal power and they may continue spending until peace becomes almost as costly as war on average. In the presence of exogenous costs of peace, the countries will occasionally spend nothing in equilibrium despite a nonempty bargaining range, resulting in a nonzero probability of war in any period. Thus, the outbreak of war through a coordination problem is inevitable in the long run.

This equilibrium, however, is not unique for all π. Consider the cases where π is close to 1—i.e., the institution of cooperation is responsive to donors—as opposed to that in which π is close to 1/2—that is, the institution of cooperation is fairly egalitarian in proposer recognition. When $\pi \approx 1$, countries deplete the bargaining surplus with diplomatic spending. When $\pi \approx 1/2$, countries can recover large gains from peace even if they spend much less than their opponent. Therefore, if π is low, a country may be willing to spend lower amounts between 0 and k if their opponent does as well. This is simply due to a stronger incentive to cooperate.

In particular, there is another equilibrium if $\pi < 2/3$ where countries always spend nonzero amounts, including amounts between 0 and k. This equilibrium cannot be sustained for larger π since the gain from cooperating as a lower spender is too small to justify low spending in equilibrium. Consider, for example, π close to 1—the value of cooperation conditional on not being recognized as proposer is very close to zero. On the other hand, with π close to 1/2, the value of cooperation conditional on not being recognized as proposer is very close to the value of cooperation conditional on being the proposer, both of which are larger than the expected war payoff. Therefore, if countries can contribute very small amounts and sustain cooperation with higher likelihood, they may prefer to even if they are very unlikely to win proposal power. The following proposition provides a formal characterization and a proof can be found in the Appendix.

Proposition 2 (MPE II). *For all $\pi < 2/3$, there is a nearly symmetric stationary MPE where each country $i = 1, 2$ plays $\sigma_i^*(p) = (r_i^*(p), \tilde{s}_i^*(p), x_i^*(p), y_i^*(x; p))$ for all $p \in P$, with $r_i^*(p)$, $x_i^*(p)$, and $y_i^*(x; p)$ as defined in Proposition 1 and $\tilde{s}_i^*(p)$ as a*

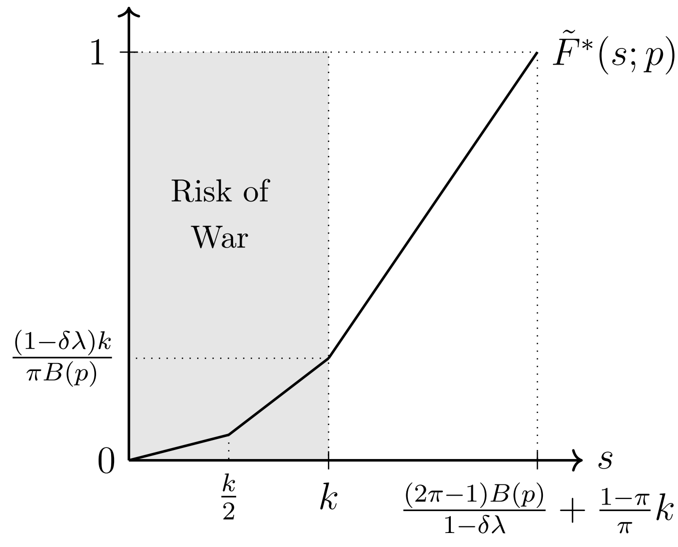

Figure 3. Equilibrium diplomatic spending according to c.d.f. $\tilde{F}^*(\cdot)$ with $\pi < \frac{2}{3}$. Now each country spends a nonzero amount with probability 1 because the gains from cooperation are large even if the country is outspent by their opponent. As $\pi \to 1/2$ from the right, each country will play a strategy arbitrarily close to mixing uniformly between 0 and k with probability $(1-\delta\lambda)2k/B(p)$ and spending k with complementary probability.

random draw from the distribution

$$\tilde{F}^*(s;p) = \begin{cases} 0 & \textit{for } s < 0 \\ \frac{(1-\delta\lambda)(2-3\pi)s}{(1-\pi)\pi B(p)} & \textit{for } s \in [0, \frac{k}{2}) \\ \frac{(1-\delta\lambda)(\pi s-(2\pi-1)k)}{(1-\pi)\pi B(p)} & \textit{for } s \in [\frac{k}{2}, k) \\ \frac{(1-\delta\lambda)(s-\frac{1-\pi}{\pi}k)}{(2\pi-1)B(p)} & \textit{for } s \in [k, \frac{(2\pi-1)B(p)}{1-\delta\lambda} + \frac{1-\pi}{\pi}k] \\ 1 & \textit{for } s > \frac{(2\pi-1)B(p)}{1-\delta\lambda} + \frac{1-\pi}{\pi}k \end{cases}$$

if $U_i(p) > W_i(p) - c_i$, *or else* $\tilde{F}^*(s;p) = 1$ *for all* $s \geq 0$.

This result is illustrated in Figure 3. While diplomatic spending in MPE II is different than in MPE I, all other behavior (whether to launch a war, what settlement to offer when proposing, and what settlements to accept if not proposing) remains the same. Though countries are always spending a positive amount, there remains a risk of war through a coordination problem since aggregate spending can still be less than k. The findings in the next section will explore both equilibria.

3 Analysis

The analysis of the game proceeds as follows. War outbreak and frequency as a function of institutional features—war outcome persistence θ and peace outcome persistence λ—as well as country patience δ is considered in §3.1. At least one country will always prefer war if they are sufficiently patient and the war outcome is sufficiently more persistent than the peace outcome. Impatient countries, on the other hand, will certainly prefer peace.

The effect of competitive diplomacy on war outbreak and frequency is considered in §3.3. Responsiveness to spending increases competition that in turn depletes the gains to be had from cooperation. In the extreme, the expected payoff from war is approximately equal to that from peace for both countries as $\pi \to 1$. Nonetheless, changes in responsiveness alone do not lead countries to initiate war.[14]

Even though countries do not choose to initiate war, war nonetheless results from a coordination problem. Ex post, at least one country will necessarily prefer to have spent zero and free-ride on the gains from cooperation than to have spent a positive amount. The result is that countries have an incentive to occasionally spend nothing in equilibrium and aggregate spending may not meet the minimum spending requirement to facilitate peace. §3.2 explores the role of key parameters on the likelihood of war from a coordination problem.

3.1 Patience and Outcome Persistence

Conventional wisdom in international politics asserts that peace can be achieved with repeated interactions of patient countries—reasoning that's inherited from the repeated Prisoners' Dilemma.[15] The results below suggest that the story is not that simple. While it is true that patient countries can prefer peace under some conditions, they may also prefer war depending on the persistence of the war outcome relative to that of the peace outcome. On the other hand, impatience guarantees that countries

[14] Given our assumption that $\pi B(p) > (1 - \delta\lambda)k$ for all $\pi \in (\frac{1}{2}, 1)$ to avoid trivial cases.

[15] The employment of the repeated Prisoners' Dilemma in the study of international cooperation dates back at least to Axelrod (1984), Keohane (1984), and Oye (1986). See Fearon (1998) for discussion of cooperation theory in the context of the bargaining model of war.

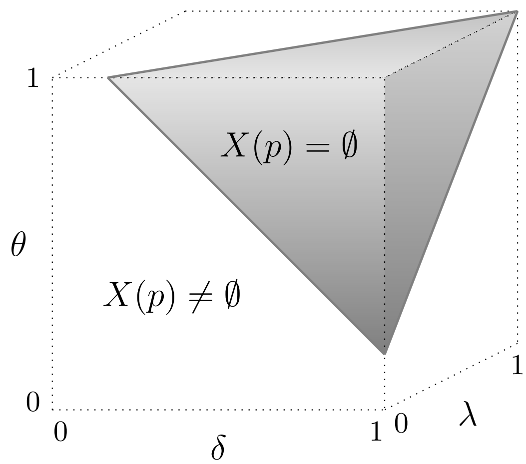

Figure 4. Parameter space $(\delta, \lambda, \theta) \in (0,1)^3$ partitioned into cases where either the bargaining range is empty, $X(p) = \emptyset$, or the bargaining range is nonempty, $X(p) \neq \emptyset$. Countries launch wars in the former case, which occurs when δ and θ are large and λ is small. Proposition 3 shows that such a partition will always divide the parameter space into two nonempty sets. Note that this illustration depicts a special case—the realized partition will rely on other features of the model, such as the costs.

prefer peace.

Proposition 3. *For all $p \in P$, there exists a $\bar{\lambda}(\delta, \theta; p) \in (0,1)$, $\bar{\theta}(\delta, \lambda; p) \in (0,1)$, and $\bar{\delta}(\theta, \lambda; p) \in (0,1)$ such that, for any $\lambda < \bar{\lambda}(\delta, \theta; p)$, $\theta > \bar{\theta}(\delta, \lambda; p)$, and $\delta > \bar{\delta}(\theta, \lambda; p)$, $W_i(p) - c_i > U_i(p)$ for at least one country $i = 1, 2$.*

Proposition 3 shows that there is always a sufficiently high level of war outcome persistence and low level of peace outcome persistence that patient countries go to war in equilibrium. An important thing to note is that it is the patient countries—not the impatient ones—that prefer to fight wars. This implies that we should take conventional "cooperation theory" wisdom via the repeated Prisoners' Dilemma with a grain of salt. If war outcomes are persistent and peaceful settlements are transient, repeated interactions among patient countries will not necessarily facilitate peace, but could instead obstruct any hope for cooperation.

When countries get increasingly patient, their preference over war or peace may not at all determined by the balance of power, but simply on the basis of outcome persistence given the costs of war and their expected diplomatic spending. This can be seen in the proof of Proposition 3 in the Appendix, where the characterization of the threshold does not rely on relative strength. In this case, $\bar{\theta}(\delta, \lambda; p) = \bar{\theta}(\delta, \lambda)$ for δ

sufficiently large. The reasoning is intuitive: if the war outcome is persistent to the extent that it is effectively permanent and peace settlements require frequent, costly bargaining, it is preferable to incur the larger costs of war once in hope to reap the rewards than to regularly pay periodic diplomatic costs for settlements that don't last. This will be true for any country regardless of their strength.

3.2 Inadvertent War

This model features a rational cause of war not present in existing crisis bargaining literature: inadvertent war as a result of an underlying coordination problem to incur costs of cooperating. Each country is willing to risk war with nonzero probability in both equilibria of the game according to their equilibrium spending strategies.

Remark 1. *$\tilde{F}^*(\cdot)$ first-order stochastically dominates $F^*(\cdot)$.*

Remark 1 implies that the probability of inadvertent war, i.e. the probability that both countries spend less than k in aggregate, will necessarily be weakly larger under MPE I than under MPE II. In fact, because we know $F^*(s;p) > \tilde{F}^*(s;p)$ only for $s < k$, we know the inequality must be strict. Denote $\phi(p)$ and $\tilde{\phi}(p)$ the probability of inadvertent war under MPE I and MPE II, respectively.

Lemma 1. *For all $p \in P$, $\phi(p) > \tilde{\phi}(p) > 0$.*

Since we have closed-form solutions for equilibrium spending strategies, we can calculate the quantities explicitly. Under MPE I, no country spends a nonzero amount less than k. Thus the probability of inadvertent war is simply given by the c.d.f. evaluated at k squared, i.e. $(F^*(k;p))^2$, or equivalently

$$\phi(p) = \left(\frac{(1-\delta\lambda)k}{\pi B(p)} \right)^2. \tag{11}$$

Under MPE II, the probability of inadvertent war is slightly more complicated. Countries now spend nonzero amounts less than k in equilibrium, implying that, conditional on a country spending an amount $s < k$, the probability of inadvertent war becomes $\tilde{F}^*(k-s;p) < \tilde{F}^*(k;p)$. We can utilize the fact that countries mix uniformly over $s \in (0, \frac{k}{2})$ and over $s \in [\frac{k}{2}, k)$ to reduce the probability of inadvertent

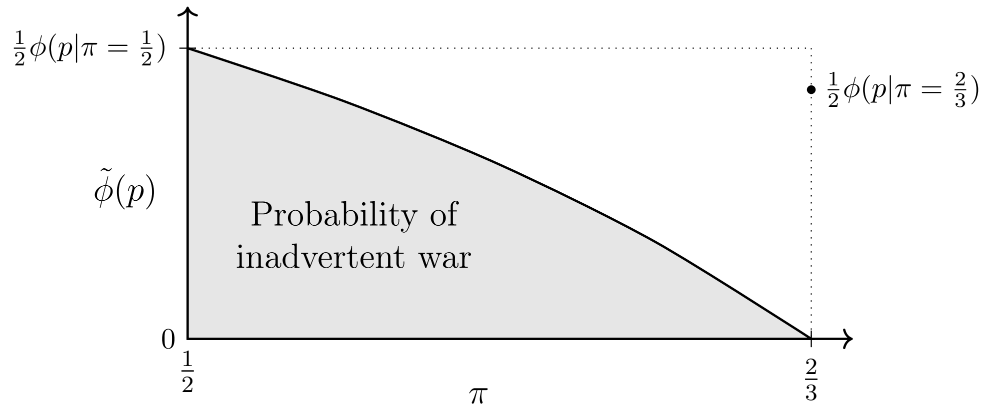

Figure 5. The probability of inadvertent war from a coordination failure under MPE II, $\tilde{\phi}(p)$, as a function of π. When π is close to $1/2$, $\tilde{\phi}(p)$ is close to $\frac{1}{2}\phi(p)$, where $\phi(p)$ is the probability of inadvertent war under MPE I. As π increases, $\tilde{\phi}(p)$ decreases, approaching zero in the limit as π approaches $2/3$ from the left. Note that $\tilde{\phi}(p)$ never reaches zero, as MPE II breaks for any $\pi \geq 2/3$. The coordinate in the upper right points out that $\phi(p)$ is also decreasing in π, though always strictly greater than $\tilde{\phi}(p)$.

war to the simple expression $\tilde{F}^*(k;p)\tilde{F}^*(\frac{k}{2};p)$, or equivalently

$$\tilde{\phi}(p) = \frac{(1-\delta\lambda)(2-3\pi)k}{2(1-\pi)\pi B(p)} \cdot \frac{(1-\delta\lambda)k}{\pi B(p)} \tag{12}$$

Together, equation (11) and (12) imply

$$\tilde{\phi}(p) = \frac{2-3\pi}{2-2\pi}\phi(p)$$

for all $p \in P$. Since $1 > (2-3\pi)/(2-2\pi) > 0$ and $\phi(p) > 0$ for all $\pi \in (\frac{1}{2}, 1)$, Lemma 1 must always hold: there is always a nonzero probability of inadvertent war and, holding π constant, this probability is strictly smaller under MPE II.

3.3 Competitive Diplomacy

The extent to which diplomatic spending is competitive affects the probability of inadvertent war. The incentive to outspend a rival country prevents trivial, cooperative pure strategy equilibria where it becomes possible to ensure peace with probability 1. For example, consider the case where diplomatic spending is not competitive. Given the existence of a bargaining surplus, we can sustain equilibria where one of the two countries spends k and the other spends nothing, always leading to peace. Additionally, without competitive spending, an equilibrium in which both countries spend $k/2$ can be sustained, also resulting in no inadvertent wars.

This does not mean that competitive diplomatic spending is always harmful—in fact, competitive spending can at times improve a country's utility. To illustrate the point, consider the case where one country always becomes the proposer if the exogenous cost to peace is paid. Then, the a simple equilibrium that results in no inadvertent wars would be one in which the proposing state incurs the cost of peace k and their opponent spends zero; however, the nonproposing country will be left indifferent with their war payoff and not enjoy any of the bargaining surplus. In the model here, competitive diplomatic spending does result in occasional inadvertent war, but both countries strictly gain from peace when a bargaining surplus exists.

Lemma 2 (Gain from peace)**.** *For all $\pi > 1/2$ and for both MPE I and MPE II, a country expects a net gain from cooperating in state p equal to*

$$\Psi(p) := \frac{(1-\pi)B(p)}{1-\delta\lambda} - \frac{1-\pi}{\pi}k.$$

For all $p \in P$, $\Psi(p) > 0$.

Lemma 2 shows that the expected gain from peace in MPE I is equal to that in MPE II—neither equilibrium is preferable ex ante even though less inadvertent war occurs under MPE II. The reason for this is that, while inadvertent war is more likely under MPE I, the expected spending is smaller. Hence it is more likely that a nonproposing country, which will not receive the benefit of the bargaining surplus, will have spent a larger amount on diplomatic spending under MPE II.

This fact simplifies welfare analysis by allowing us to focus on both equilibria simultaneously. A natural question that arises out of the competitive diplomacy in this game is: does competition improve or harm country welfare?

Proposition 4 (Optimal π)**.** *The level of responsiveness $\pi \in (\frac{1}{2}, 1)$ that maximizes country welfare is given implicitly by*

$$\pi^*(p) := \sqrt{\frac{(1-\delta\lambda)k}{B(p)}}$$

when $((B(p))^{-1}(1-\delta\lambda)k)^{1/2} > \frac{1}{2}$*. Otherwise, for any* $\pi \in (\frac{1}{2}, 1)$*, there always exists a* $\pi' \in (\frac{1}{2}, \pi)$ *that improves welfare.*

If countries are both very patient and peace settlements are persistent, then countries fare better with a very small π close to 1/2. On the other hand, if either countries are impatient or peace settlements are transient, the optimal π will be an interior value. Some competition is useful for improving country welfare. Since $\phi(p)$ and $\tilde{\phi}(p)$ are both decreasing in π, this implies that optimizing country welfare is not equivalent to reducing the probability of inadvertent wars.

4 Discussion

In this paper, I develop a dynamic crisis bargaining model in which countries decide to fight or cooperate in a setting where peace is costly and diplomatic spending is rewarded with bargaining leverage. I establish that patient countries will prefer to fight wars if the war outcome is sufficiently persistent and peace agreements are transient. Further, even if a bargaining range does exist, competition over the bargaining surplus erodes the net gain from cooperation. In equilibrium, each country spends below the diplomatic spending threshold with nonzero probability due to an underlying coordination problem, leading to inadvertent wars. The frequency of inadvertent war relies directly on parameters of the model, including the persistence parameters, the features of institutional cooperation, and the shadow of the future.

The results of this model have direct implications on the outbreak and frequency of crises. Persistence parameters θ and λ can be understood as a reduced form of some larger game happening in the international order. In this sense, a small θ might reflect the expectations of a territory-seeking country when a third, unmodeled superpower has credibly stated they intend to intervene and expropriate any territory conquered in war. On the other hand, a large θ may reflect the fact that no other country is willing to intervene after territory is won in war and the international norm is to uphold the status quo. The same reasoning could be applied to λ, with small λ reflecting low enforcement capability and high λ reflecting high enforcement capability. Understanding these differences—e.g. when the United States is willing to intervene to upend an outcome that resulted from war and when they are not—can speak to which regions of the world are most prone to conflict.

An interesting feature of this model that is outside of the scope of this study is the role of information. In this model, the results are achieved with players that are perfectly informed. Consider, however, an extension of the model where countries underestimate their opponent's costs of war, which correspondingly leads them to overestimate the value of the bargaining range. Prior to an offer being extended, a country would observe the amount their opponent spent on diplomacy and update their beliefs about the bargaining range accordingly. Learning from diplomatic endeavors may therefore lead to better offers and less war in equilibrium compared to the standard crisis bargaining model.

There may be institutional features not included in the model that can mitigate the coordination problem. Consider, for example, favoritism in proposer recognition such that country 1 is disadvantaged. Under this biased institution, country 2 may still be recognized with higher probability even when they spend less than country 1, possibly providing 2 with enough incentive to always contribute at least k to ensure cooperation. If so, a consequence may be that country 1 spends zero with higher probability and rarely enjoys the bargaining surplus. Thus, a biased institution may be more effective at preventing inadvertent war from a coordination problem, but it should also reduce the incentive to cooperate for at least one country.

Lastly, it would be interesting to explore this dynamic with an intelligent institution that has agency and preferences of its own. For example, instead of an exogenous threshold cost for cooperation, we might consider institutions that optimally choose k in order to achieve their ends, which would likely be some combination of peace facilitation and profit maximization. An analysis like this, while outside the scope of this paper, would likely shed useful insight on agency problems of international organizations in the context of crisis bargaining.

Appendix

Proof of Proposition 1. The war decision is straightforward. The countries seek to maximize their expected utility over the long run and if, given state $p \in P$, their war continuation value $W_i(p)$ less the costs of war c_i is larger than their continuation value from cooperating, $V_i(p)$, they will necessarily prefer to fight.

Then, if there exists a bargaining surplus, $B(p) = x_1^*(p) - x_2^*(p) > 0$, both countries will cooperate and choose an amount to spend as a function of the state p. Then, a country i that's recognized as proposer will receive the value of the bargaining surplus today and possibly in the future, with likelihood according to λ. Hence, the expected net gain of winning proposal power becomes

$$B(p) + \delta\lambda B(p) + (\delta\lambda)^2 B(p) + \ldots = \frac{B(p)}{1-\delta\lambda} \tag{A1}$$

However, the net gain from becoming proposer is equal to the net gain from spending more on diplomacy than your opponent divided by recognition probability π. Hence, the expected net gain from spending an amount s on diplomacy is

$$\frac{\pi B(p)}{1-\delta\lambda}\Pr(s > s_{-i}) + \frac{(1-\pi)B(p)}{1-\delta\lambda}\Pr(s_{-i} > s) - s. \tag{A2}$$

Clearly, the net gain is zero when $B(p) = 0$, in which case neither country will be willing to spend a positive amount in equilibrium, leading to war as a result of the unsatisfied minimum funding requirement k. Therefore, assume $B(p) > 0$.

It is straightforward that there will be no pure strategy spending in equilibrium. If a country always spend some amount $s > 0$, their opponent would either deviate to an amount greater than s or zero. If the opponent deviated to zero, the country will prefer to spend less than s. If the opponent deviated to an amount greater than s, the country will prefer to move to a greater amount or deviate to zero. If both countries spend the same amount, they will either have an incentive to increase their spending a marginal amount to increase their gain by approximately double or they will prefer to deviate to zero. This is standard in common value all-pay contests.

Therefore, I proceed by looking for a mixed strategy given by cumulative distribution function (c.d.f.) $F^*(\cdot)$ that satisfies equation (A2) for both countries. Note that, because $B(p) > 0$, there is a strict gain to cooperating that is decreasing in π. In particular, the expected utility of country i spending $s = 0$ is strictly greater than their war payoff. We can write the payoff from spending zero as

$$W_i(p) - c_i + \frac{(1-\pi)B(p)}{1-\delta\lambda}(1 - F^*(k)). \tag{A3}$$

Given a country is spending zero with some positive probability, their opponent has incentive to spend at least k with positive probability, which yields an expected payoff

$$W_i(p) - c_i + \frac{\pi B(p)}{1-\delta\lambda}F^*(k) + \frac{(1-\pi)B(p)}{1-\delta\lambda}(1 - F^*(k)) - k. \tag{A4}$$

Using equations (A3) and (A4), we can solve for $F^*(k)$,

$$F^*(k) = \frac{(1-\delta\lambda)k}{\pi B(p)}. \tag{A5}$$

We know that k cannot be the top of the support because $\pi B(p) > (1-\delta\lambda)k$ by assumption and they can get a strictly higher payoff by contributing slightly more than k to get recognized as proposer. Then, a country i spending $s > k$ will receive an expected payoff

$$W_i(p) - c_i + \frac{\pi B(p)}{1-\delta\lambda}F^*(s) + \frac{(1-\pi)B(p)}{1-\delta\lambda}(1 - F^*(s)) - s. \tag{A6}$$

Using the indifference condition for equations (A4) and (A6) and plugging in equation (A5), we find that for $s \geq k$

$$F^*(s) = \frac{(1-\delta\lambda)(s - \frac{1-\pi}{\pi}k)}{(2\pi-1)B(p)}. \tag{A7}$$

By definition of a c.d.f., we know the largest amount a country can spend and still be indifferent is given by $\bar{s} := \inf\{s \geq 0 : F^*(\bar{s}) = 1\}$. Using equation (A7), we find that

$$\bar{s} = \frac{(2\pi-1)B(p)}{1-\delta\lambda} + \frac{1-\pi}{\pi}k. \tag{A8}$$

These equations yield an equilibrium spending strategy presented in Proposition 1. To check for profitable deviations, consider the case where a country spends $s > \bar{s}$

with nonzero probability. By deviating, their payoff will be

$$W_i(p) - c_i + \frac{\pi B(p)}{1-\delta\lambda} - s < W_i(p) - c_i + \frac{\pi B(p)}{1-\delta\lambda} - \bar{s}, \tag{A9}$$

and therefore they do not deviate. In words, a country already wins with certainty when spending $\bar{s}$, so there is no reason to ever spend more given their opponent plays this strategy as well.

Further, consider a deviation to spending a nonzero amount less than k, $s \in (0, k)$, with some probability. By deviating, their expected payoff will be

$$W_i(p) - c_i + \frac{(1-\pi)B(p)}{1-\delta\lambda}(1 - F^*(k-s)) - s. \tag{A10}$$

Since their opponent is playing a strategy such that $F^*(k) = F^*(k-s)$, we can plug this into equation (A10) and see that their payoff is equal to

$$W_i(p) - c_i + \frac{(1-\pi)B(p)}{1-\delta\lambda}(1 - F^*(k)) - s \tag{A11}$$

which is strictly less than their payoff from spending zero given by equation (A3), hence this is not a profitable deviation. This is sufficient to show that c.d.f. $F^*(\cdot)$ is an equilibrium spending strategy.

Now consider a country's offer. From the main text, equations (5) and (6) yield implicit conditions for $\overline{x}(p)$ and $\underline{x}(p)$, which represent the settlements at which country 1 and 2 are left indifferent between peace and war, respectively. If this value is in the unit interval, this is the country's equilibrium offer as offering a settlement less favorable to their opponent will result in a rejection and subsequent war, whereas offering a settlement more favorable to their opponent will result in acceptance but a worse settlement for themselves. If these values are not in the unit interval, the countries offer the closest value in the unit interval by Euclidean distance. In the event this results in acceptance, this is the best the country can do and they will consume the entire pie by settlement. If this results in rejection, all other possible settlement offers would likewise result in rejection, so there is no profitable deviation.

Their decision to accept or reject and offer is also straightforward. At the point at which a proposal has been made, a proposer has already been recognized and spending has already occurred, hence diplomatic spending in that period becomes a sunk cost. Therefore, when an offer x is proposed, countries decide whether they prefer $U_i(x;p) = U_i(p) + s_i(p)$ or their war continuation value and accept or reject accordingly. $\square$

Proof of Proposition 2. To look for a new equilibrium diplomatic spending strategy $\tilde{F}^*(\cdot) \neq F^*(\cdot)$ such that countries occasionally spend positive amounts less than k, suppose there exists an $s \in (0, \frac{k}{2})$ such that $\tilde{F}^*(s) \neq \tilde{F}^*(s+\varepsilon)$ for any $\varepsilon \neq 0$. Then we know that

$$W_i(p) - c_i + \frac{(1-\pi)B(p)}{1-\delta\lambda}(1-\tilde{F}^*(s')) - s = W_i(p) - c_i + \frac{\pi B(p)}{1-\delta\lambda} - \bar{\bar{s}} \tag{A12}$$

where $\bar{\bar{s}} := \inf\{s \geq 0 : F^*(s) = 1\}$ and $s' := k - s$. Solving for $\tilde{F}^*(s')$ yields

$$\tilde{F}^*(s') = \frac{(1-\delta\lambda)(\bar{\bar{s}} + s' - k)}{(1-\pi)B(p)} - \frac{2\pi - 1}{1-\pi}. \tag{A13}$$

We also know that in equilibrium

$$\begin{aligned} & W_i(p) - c_i + \frac{(1-\pi)B(p)}{1-\delta\lambda}(1-\tilde{F}^*(s')) - s \\ = {} & W_i(p) - c_i + \frac{(1-\pi)B(p)}{1-\delta\lambda}(1-\tilde{F}^*(s')) + \frac{\pi B(p)}{1-\delta\lambda}(\tilde{F}^*(s') - \tilde{F}^*(s)) - s' \end{aligned} \tag{A14}$$

which implies

$$\tilde{F}^*(s') = \tilde{F}^*(s) + \frac{(1-\delta\lambda)(s'-s)}{\pi B(p)}. \tag{A15}$$

Using equations (A13) and (A15) we can solve for $F^*(s)$,

$$\tilde{F}^*(s) = \frac{(1-\delta\lambda)(\pi\bar{\bar{s}} + (2-3\pi)s - (1-\pi)k)}{(1-\pi)\pi B(p)} - \frac{2\pi-1}{1-\pi}. \tag{A16}$$

To proceed we need to know the value of $\bar{\bar{s}}$. We can find it from the equilibrium condition that for any $s \geq k$, we have

$$W_i(p) - c_i + \frac{(1-\pi)B(p)}{1-\delta\lambda}(1 - \tilde{F}^*(s)) + \frac{\pi B(p)}{1-\delta\lambda}\tilde{F}^*(s) - s \tag{A17}$$
$$= W_i(p) - c_i + \frac{(1-\pi)B(p)}{1-\delta\lambda}(1 - \tilde{F}^*(k))$$

where the righthand side follows from the need to have zero in the support. If zero is not in support, there is always a profitable deviation from the infinum of the support to zero. We know from equation (A13) that $F^*(k) = (\pi B(p))^{-1}(1-\delta\lambda)k$, so we can solve the above equation to recover

$$\bar{\tilde{s}} = \frac{(2\pi - 1)B(p)}{1-\delta\lambda} + \frac{1-\pi}{\pi}k. \tag{A18}$$

Plugging this into equation (A16), we have that for any $s \in (0, \frac{k}{2})$,

$$\tilde{F}^*(s) = \frac{(1-\delta\lambda)(2-3\pi)s}{(1-\pi)\pi B(p)}. \tag{A19}$$

For this to be a valid c.d.f., we require $(2-3\pi)s > 0$, or $\pi < \frac{2}{3}$. So for sufficiently high values of π, this is not possible. Given $\pi < \frac{2}{3}$, suppose there exists an $s' \in (\frac{k}{2}, k)$ such that $\tilde{F}^*(s') \neq \tilde{F}^*(s' + \varepsilon)$ for any $\varepsilon \neq 0$. Then, letting $s = k - s'$, we have

$$\tilde{F}^*(s') = \frac{(1-\delta\lambda)(\pi s' + (1-2\pi)k)}{(1-\pi)\pi B(p)} \tag{A20}$$

For this expression to be a valid c.d.f. we require $\pi > \frac{k}{2k+s'}$, which should always hold for any $\pi \in (\frac{1}{2}, \frac{2}{3})$. By equations (A18) and (A20), we know that $\bar{\tilde{s}} = \bar{s}$ and $\tilde{F}^*(k) = F^*(k)$, which implies $\tilde{F}^*(s) = F^*(s)$ for all $s \geq k$.

There is no profitable deviation from $\tilde{F}^*(\cdot)$ for the same reasons as $F^*(\cdot)$. Countries are indifferent over spending at all amounts in the support and the country would do strictly worse by spending more than the supremum of the support. Moreover, given equilibrium spending $\tilde{s}_i(p)$ as a random draw from $\tilde{F}^*(\cdot)$, there is no profitable deviation from other actions for the same reasons as Proposition 1. □

Proof of Proposition 3. For contradiction, suppose $U_i(p) \geq W_i(p) - c_i$ for both $i = 1, 2$ for all $\theta \in (0, 1)$, $\lambda \in (0, 1)$, and $\delta \in (0, 1)$. Let any $x \in [0, 1]$ be the expected peaceful settlement in state p, under which the continuation values for peace are

$$U_1(p) = x - \int_0^\infty s dF_1^*(s;p) + \frac{\delta}{1-\delta\lambda}\left[\lambda x + (1-\lambda)\sum_{p' \in P} V_1(p')q(p'|p,a)\right]$$

and

$$U_2(p) = 1 - x - \int_0^\infty s dF_2^*(s;p) + \frac{\delta}{1-\delta\lambda}\left[\lambda(1-x) + (1-\lambda)\sum_{p' \in P} V_2(p')q(p'|p,a)\right].$$

Note that $\int_0^\infty s dF_i^*(s;p)$ relies on equilibrium behavior as a function of state p, but it is sufficient for the argument that follows to assume that $\int_0^\infty s dF_i^*(s;p) > s_i$ for some fixed and arbitrarily small $s_i > 0$. If the proof holds under such an s_i, it must also hold under true expected spending in equilibrium since $U_i(p)$ is decreasing in spending and expected spending must be greater than zero to facilitate peace.

Further, we only need to look for one-shot deviations, so we can assume that $V_i(p) = U_i(p)$ for all $p \in P$ and denote $U_i(\bar{p}) := \sum_{p' \in P} U_i(p')q(p'|p,a)$. Note that if $U_1(\bar{p}) > U_1(p)$, then necessarily $U_2(\bar{p}) < U_2(p)$. Thus if we can show there must exist thresholds that make $W_i(p) - c_i > U_i(p)$ for both i when $U_i(p) = U_i(\bar{p})$, it must also hold for at least one country when $U_i(p) \neq U_i(\bar{p})$. Therefore, assume $U_i(p) = U_i(\bar{p})$ and look to show there exists a δ, θ, and λ such that both countries prefer war.

With this, we can rewrite continuation values as

$$U_1(p) = \frac{x - (1-\delta\lambda)s_1}{1-\delta}$$

and

$$U_2(p) = \frac{1 - x - (1-\delta\lambda)s_2}{1-\delta}.$$

We need to compare this to their war continuation values. Given our assumption that $\sum_{p' \in P} V_i(p')q(p'|p,a) = U_i(p)$, we can plug these into $W_1(p)$ and $W_2(p)$ to recover

$$W_1(p) = \frac{p}{1-\delta\theta} + \frac{\delta(1-\theta)}{1-\delta\theta} \cdot \frac{x-(1-\delta\lambda)s_1}{1-\delta}$$

and

$$W_2(p) = \frac{1-p}{1-\delta\theta} + \frac{\delta(1-\theta)}{1-\delta\theta} \cdot \frac{1-x-(1-\delta\lambda)s_2}{1-\delta}.$$

This implies that $W_1(p) - c_1 > U_1(p)$ and $W_2(p) - c_1 > U_2(p)$ if and only if

$$p - (1-\delta\theta)c_1 + (1-\delta\lambda)s_1 > p + (1-\delta\theta)c_2 - (1-\delta\lambda)s_2$$

Analogously, there does not exist an $x \in [0,1]$ that satisfies either country if

$$\theta > \bar{\theta}(\delta, \lambda) := \frac{c_1 + c_2 - (1-\delta\lambda)(s_1+s_2)}{\delta(c_1+c_2)}.$$

Here $\bar{\theta}(\cdot)$ is not a function of the state p because we took an s_1 and s_2 strictly less than the expected equilibrium spending in a given state p; however, in general, $\bar{\theta}(\cdot)$ will rely on the state p since equilibrium spending will rely on the state p. It is sufficient to show that $\bar{\theta}(\delta, \lambda) < 1$ for sufficiently high δ and sufficiently low λ. Note that $\bar{\theta}(\delta, \lambda) < 1$ when $(s_1+s_2)/(c_1+c_2) > (1-\delta)/(1-\delta\lambda)$. The left-hand side is a positive real number that does not rely on parameters, whereas the multivariable limit of the right-hand side is

$$\lim_{(\delta,\lambda)\to(1,0)} \frac{1-\delta}{1-\delta\lambda} = 0.$$

Hence the expression is satisfied in the limit of (δ, λ), completing the proof. □

Proof of Lemma 2. For MPE I, this lemma follows directly from the expected payoff in peace given by equation (A3) less war payoff $W_i(p) - c_i$, which is equal to

$$\frac{(1-\pi)B(p)}{1-\delta\lambda}(1-F^*(k)) \tag{A21}$$

Plugging in $F^*(k)$ defined by (A5) this becomes

$$\frac{(1-\pi)B(p)}{1-\delta\lambda} - \frac{1-\pi}{\pi}k. \tag{A22}$$

For MPE II, the condition is the same since $\tilde{F}^*(k) = F^*(k)$. □

Comments on Remark 1. Recall that $\tilde{F}^*(\cdot)$ first-order stochastically dominates $F^*(\cdot)$ if and only if $F^*(s) \geq \tilde{F}^*(s)$ for all s and $F^*(s) > \tilde{F}^*(s)$ for some s. Then, note that $\tilde{F}^*(s) = F^*(s)$ for all $s \notin (0, k)$. For all $s \in [\frac{k}{2}, k)$,

$$F^*(s) = \frac{(1-\delta\lambda)k}{\pi B(p)} > \frac{(1-\delta\lambda)(\pi s - (2\pi - 1)k}{(1-\pi)\pi B(p)} = \tilde{F}^*(s)$$

and for all $s \in (0, \frac{k}{2})$,

$$F^*(s) = \frac{(1-\delta\lambda)k}{\pi B(p)} > \frac{(1-\delta\lambda)(2-3\pi)s}{(1-\pi)\pi B(p)} = \tilde{F}^*(s).$$

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