# Some for the Price of One: Vote Buying on a Network

Perry Carter Joseph J. Ruggiero

March 31, 2022

**Abstract**

Empirical studies of vote buying and clientelism emphasize the importance of social networks and local knowledge, but formal analyses of the candidates' problem have not examined these factors. Moreover, existing models typically abstract away from policy considerations. We provide a formal model where candidates may choose to employ vote buying on a network of policy-motivated voters at the expense of a public good in order to improve their chances of election. In addition to voter and candidate preferences, we find an important role for social structure, office motivations, and the extent of inefficiencies in public good provision. In particular, it is possible to sustain equilibria in which no candidate will pursue vote buying strategies even in the absence of constraining institutions or social norms.

---

Carter: Ph.D. Candidate in Politics, Princeton University. pjcarter@princeton.edu.
Ruggiero: Ph.D. Candidate in Politics, Princeton University. jruggiero@princeton.edu.

We thank Kris Ramsay, Gleason Judd, Matias Iaryczower, Germán Gieczewski, and the participants of the Princeton University Political Economy Research Seminar for their helpful comments and suggestions.

# Contents



# 1 Introduction

It is widely accepted that social structure affects the distribution of private political gains, yet formal analyses of vote buying do not account for interdependence between actors. At the same time, a growing literature in economics and graph theory provides a technical framework for handling strategic interactions on dense social networks, but has thus far seen only limited applications in political science.

This article joins these approaches, analyzing a networked model of a large election in which candidates compete to influence policy-motivated voters through private transfers—bribes—that come at the expense of a public good. By combining the techniques of random graph analysis with the insights from a series of papers that have observed that solutions to a broad class of games on networks correspond the vector of Katz-Bonacich centralities (Ballester, Calvó-Armengol, and Zenou 2006;

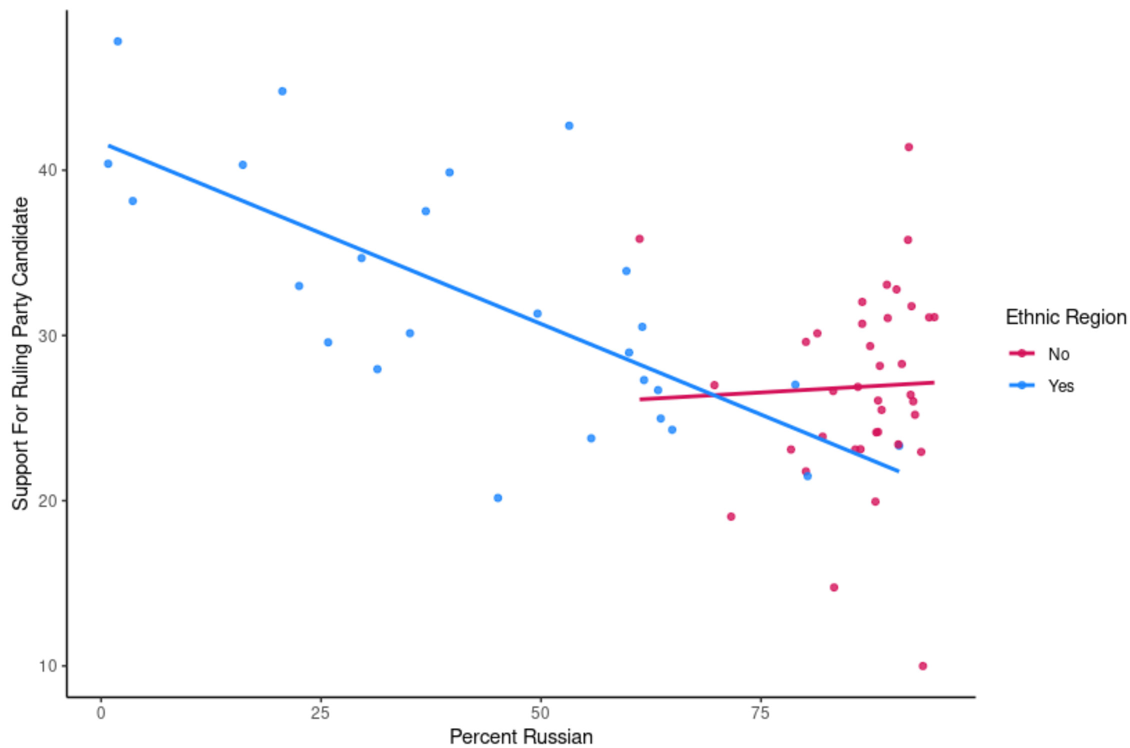


Figure 1: Electoral Performance of Incumbents in 2016 Russian Elections as a Function of Regional Ethnic Makeup

Battaglini and Patacchini 2018), we are able to derive sharp results characterizing the equilibrium distribution of bribes while retaining much of the rich complexity of the setting.

In particular, our analytical framework allows us to overcome a major limitation of many network models: the need to begin by taking as granted a highly complex discrete graph structure. By focusing instead on the underlying generative process that gives rise to observed social structure, we are able to derive closed-form expressions for the dependence of equilibrium strategies on deep features of society, such as homophily and density, that hold with probability approaching unity in large societies.

In order to demonstrate the value of this approach, we begin by briefly considering an illustrative example that highlights the limitations of existing theory in accounting for dependence on network structure. Figure 1 depicts performance in the 2016 elections to the Russian state Duma (the lower chamber of parliament), which, while neither free nor fair, nonetheless exhibited considerable variation in the performance of candidates from the ruling United Russia (UR) Party. In particular, despite UR's increasing association with ethnic Russian nationalism and policies of centralization, the opposite demographic pattern emerges: on average, a five percentage point increase in the proportion of ethnic Russians in an electoral district is associated with a one percentage point *decrease* in the ruling party's vote share.

Scholars have consistently explained this apparent puzzle by reference to the dense ethnic networks that characterize majority-minority regions and their ability to facilitate machine politics and vote buying (Drobizheva 1999; Hale 2003; Golosov 2011; Sharafutdinova 2013; White and Saikkonen 2017). Despite this consensus over the primary explanatory role of social structure, however, no comparative work has systematically examined it, and theoretical accounts rely primarily on the concept of "density," which may not well characterize empirical differences across social networks (Larson and Lewis 2017). Indeed, given the extreme complexity of networks, precise predictions are impossible to generate without an explicit model of social interactions such as the one presented in this paper.

The model may also be useful for other applications in political science. For example, there has been an active literature in international political economy on vote buying in the United Nations Security Council (UNSC). Thacker (1999) argues that alignment with the United States in the United Nations General Assembly increases the probability of receiving a loan from the International Monetary Fund (IMF). Vreeland and Dreher (2014) provide further evidence that powerful countries trade money for political favors with Security Council members. Voting alignment in the UNSC with the United States, for example, is shown to be associated with greater financial assistance through direct foreign aid, as well (Alexander and Rooney 2019).

## 1.1 Vote Buying and Social Influence

Vote buying—distinguished from other forms of clientelist linkages (Kitschelt 2000) in that benefits accrue directly to the individual voter and that the exchange is temporally confined to the period surrounding the election (Schaffer and Schedler 2007)—is, despite its universal illegality, a prevalent feature of younger democracies and is associated with a variety of negative economic, social, and institutional outcomes (Keefer 2007; Hicken 2011).

Despite the potential effectiveness of private inducements as an electoral strategy in the absence of strong partisan attachments, however, prospective vote buyers face several key challenges in implementing these strategies in large elections. The first and most extensively studied problem is that of commitment—since the whole exchange is illegal and ballots are typically secret, politicians cannot be certain that voters will follow through on their promise, nor can voters be sure that politicians will deliver on any promise of goods if elected (Brusco, Nazareno, and Stokes 2004; Nichter 2008; Finan and Schechter 2012; Keefer and Vlaicu 2017).

This problem, nevertheless, is frequently overcome in practice, principally through a combination of normative commitment to reciprocity and the integration of voters into dense social networks that facilitate information flows and two-way monitoring (Stokes 2005; Nichter 2008; Finan and Schechter 2012; Calvo and Murillo 2013; Frye, Reuter, and Szakonyi 2019). Stable connections to voters thus allow politicians to simultaneously cultivate social norms that are conducive to successful vote buying and to gather information on their actual vote choice. In addition, connections *between* voters may play a crucial role, as social pressure and peer-monitoring make highly-connected voters more likely to follow through on a vote buying agreement (Cruz

2019).

A second difficulty relates to the identification of voters and the effective delivery of benefits to them. In large elections, no candidate is likely to have enough information, resources, or bureaucratic capacity to reliably identify swing voters, approach them individually, and provide private goods in a timely and cost-efficient manner. Here, again, the cultivation of partisan political networks appears to be key, relaying information to politicians and providing established distribution channels (Stokes 2005; Calvo and Murillo 2013). This also creates a general advantage in clientelist structures for those voters with pre-existing ties, such as family relationships, to candidates, as they are more likely to be targeted for high-value goods (Fafchamps and Labonne 2017; Cruz, Labonne, and Querubin 2017).

A final challenge, which increases in magnitude with the size of the election, is the imbalance between the resources available for patronage and the number of votes that needs to be won. While high-credibility goods such as public-sector jobs or political favors are by far the most effective at solving the commitment problem (Robinson and Verdier 2013; Oliveros 2016), the supply of such goods is unlikely to be sufficient to win even local elections that are decided by voting margins of several thousand. Moreover, in many developing countries candidates lack the financial and organizational resources necessary to establish the kind of monitoring machines highlighted above, exacerbating the problem by making monitoring essentially impossible (Kramon 2016*a*; Guardado and Wantchekon 2018). Hence, a growing literature has begun to recognize another dimension of vote buying as an electoral strategy: handouts may serve to provide voters with *information* about the likely behavior of the candidate once in office (Kramon 2016*a*,*b*; Auerbach and Thachil 2018; Auerbach 2019).

Thus, a consistent theme in studies of clientelism in general, and vote buying in particular, is that network linkages that connect elites to voters and voters to one another are crucial determinants of the efficacy of these electoral strategies. However, the specific features of social networks that are most conducive to vote buying, as well as the types of voters most likely to be targeted, remain largely unspecified. For instance, while a key conclusion of the model presented in Stokes (2005, p. 318) is that clientelist parties are "effective to the extent that they insert themselves into the social networks of constituents," the actual interaction between candidates and voters is modeled as an iterated Prisoners' Dilemma, yielding no clear predictions regarding the consequences of variation in "insertion."

Another important example of this difficulty can be found in the closely related literature on the political economy of ethnicity, where the "density" of ethnic groups is taken to be one of their defining features, and acts as a key mediating variable between diversity and outcomes such as conflict, public goods provision, and the development of clientelism (Putnam 2000; Miguel and Gugerty 2005; Chandra 2007; Habyarimana, Humphreys, Posner, and Weinstein 2009; Gubler and Selway 2012). One of the most influential formal statements of this perspective comes from Fearon and Laitin (1996), who study a model explaining the prevalence of cooperative equilibria in diverse societies by the relatively high probability of interaction among members of the same group, rather than between groups. Although this high probability is interpreted as a consequence of the density of in-group networks, connections between individual agents are not explicitly modeled, making it difficult to derive precise predictions about how variations in social structure affect the likelihood of cooperation. Indeed, in an empirical analysis of the features of actual ethnic networks in two Ugandan villages, Larson and Lewis (2017) find the opposite: network density is positively correlated with diversity and, contrary to theoretical expectations, including conventional predictions from social network theory regarding the "strength of weak ties" (Granovetter 1973) has negative consequences for information spread. While data on politically relevant social networks at this level of detail remain sparse, this example highlights the need to engage explicitly with network structure in order to generate empirically valid comparative predictions.

Given the inherent complexity of social networks, this is a task that would seem well-suited to a formal modeling approach. Few models of vote buying, however, have considered the role of connections between players. Indeed, many of the most influential models of vote buying assume continuous distributions of voters (Groseclose and Snyder 1996; Lizzeri and Persico 2001), for which results do not necessarily generalize to finite populations (Banks 2000; Dekel, Jackson, and Wolinsky 2008). While this literature has generated important insights into the possibility of vote buying to induce inefficient supermajority coalitions (Groseclose and Snyder 1996; Banks 2000), the difficulty of overcoming private incentives by providing public goods (Lizzeri and Persico 2001), the role of varying commitment structures and institutions in mitigating the inefficiencies introduced by vote buying (Dal Bó 2007; Dekel, Jackson, and Wolinsky 2008), and the institutional factors driving the mix of strategies chosen by clientelist machines (Gans-Morse, Mazzuca, and Nichter 2014), variation is driven either by individual factors (preferences) or by macro-level variation in institutional environments. Despite the prominent role afforded to ties *between* actors in empirical accounts, however, this aspect of the strategic environment has gone largely unexplored.

An important exception, however, comes from Battaglini and Patacchini (2018), who study the analogous problem of influencing members of a legislature through campaign contributions using a network model similar to the one considered in this paper.[1] The major finding of this work is that the equilibrium transfers to voters (legislators) are proportional to their Katz-Bonacich centrality (Bonacich 1987), weighted by the equilibrium probability of pivotality. While the elegance of this result, an ana-

[1] See also Battaglini, Sciabolazza, and Patacchini (2020).

logue of which is presented in the next section, is remarkable, both the structure of the game and the equilibrium strategies (which require the inversion of a matrix of arbitrary size) remain highly complex objects, making it essentially impossible to derive useful comparative statics. Since, given an arbitrary graph on $n$ vertices, there are $2^{-1}n(n-1)$ possible undirected edges, the number of possible alterations to be considered increases rapidly with the size of the voting population. When combined with the fact that the addition or removal of a single edge could dramatically alter the equilibrium to *all* agents, this largely precludes any analysis of the dependence of strategies on the underlying social structure.

In order to resolve this difficulty, we draw on recent advances in the analysis of random graphs, which make it possible to draw sharp conclusions about the effects of social structure by shifting attention from realized networks to an underlying generative model. We briefly review the key results in the following section, before presenting the model.

## 1.2 Random Network Models

An immediate problem that frequently arises in the analysis of economic games played on complex networks is that equilibrium strategies, even when unique, are frequently defined only implicitly and grow combinatorically in complexity with the size of the network. In order to avoid this difficulty, we rely on a now well-established result from Ballester, Calvó-Armengol, and Zenou (2006), who show that a broad class of games can be studied as a network game where the Nash equilibrium strategies are proportional to the Katz-Bonacich centrality of agents (Katz 1953; Bonacich 1987). This approach facilitated the analysis of settings as diverse as peer effects in education (Calvó-Armengol, Patacchini, and Zenou 2009) and co-sponsorship networks in the U.S. Congress (Battaglini and Patacchini 2018; Battaglini, Sciabolazza, and Patacchini 2020), providing a good empirical fit when compared to alternative measures of centrality from the social networks literature.

Despite the appealing parsimony of this approach, however, the sensitivity of any agent's Katz-Bonacich centrality to small changes elsewhere on the network makes analysis of the role played by the underlying structural features of the network exceedingly difficult. In order to overcome this problem, we adopt an approach that has recently gained in popularity in the literature on learning and information diffusion (Board and Meyer-ter Vehn 2021), eschewing consideration of exact graphs in favor of a random generative model that shares the same features of interest, such as density or homophily. In this regard, important recent work by Dasaratha (2020) and Mostagir and Siderius (2021) provides a framework for studying the dependence of centrality measures on social structure based on the observation that in large networks these measures are, with high probability, close to their values in an appropriately-defined "average" network.

Having derived a novel measure of centrality as the unique equilibrium of the model presented in the next section, we prove analogous results that permit us to study how the efficacy of vote buying and equilibrium transfers depend on social structure. By extending several key results in the spectral theory of random graphs (Chung and Radcliffe 2011)[2] to the case of weighted normalized graphs, in Section 3.1 we derive mild conditions on the behavior of social ties in the expected network as the number of voters grows that ensure that the exact network can be replaced with its average counterpart without loss. By doing so, we are able to derive closed-form algebraic expressions for the equilibrium bribes of each voter, generating unusually sharp comparative statics.

[2] See Chung and Graham (1997) for a review of the mathematical foundations of spectral graph theory.

## 2 Model

We begin by assuming that voters care about policy in a unidimensional space and the provision of a public good, but they can also be "bribed" with private transfers that influence their likelihood of voting for one candidate over another. Since we are primarily interested in elections where $n$ is sufficiently large that the probability of pivotality is approximately zero, we assume expressive voting based on net preference after bribes.

In order to retain our focus on the network-specific elements of the model, we additionally assume that both candidates are endowed with full commitment power, so that bribes can be treated as either up-front or as campaign promises without consequence (Dekel, Jackson, and Wolinsky 2008). Bribes are not a binding contract, however, voters may receive transfers from both candidates, but will ultimately vote in accordance with their own preferences. As such, neither candidate can ever be certain that a vote has been "bought," reflecting the commitment problem highlighted in much of the literature.

The key feature of this model is the nature of network dependence. In addition to being influenced by direct transfers and campaign promises, voters place some weight on the likelihood of each of their neighbors on the network for voting for a given candidate when making their own decision. Modeling network spillovers in this way allows us to capture the main features of the strategic situation identified in the literature while retaining a tractable model capable of generating clear predictions.

Substantively, this mechanism has two main interpretations. First, voters can be thought of as *communicating* with their acquaintances about their intent to vote, which provides information about the candidate's desirability, such as ability to provide voters with employment opportunities. Providing a transfer to a given voter therefore also increases the likelihood of their connections supporting that candidate since they hold more favorable posteriors, consistent with empirical research highlighting the informational role of vote buying (Kramon 2016*b*). Importantly, spillovers are not limited to a voter's direct connections. Since these direct contacts will have updated their beliefs, they then communicate these to *their* connections, and so on. Thus, a bribe paid to *any* voter on the network will positively impact *all* voter's likelihood of voting for a candidate, albeit with diminishing returns in social distance.

A second interpretation of social spillovers is as a consequence of social pressure. Even if voters do not gain any relevant information from their neighbors, they may still be intrinsically motivated to take the same action as a majority of them. Common mechanisms underlying this are a desire for conformity—voters may simply derive utility from doing the same thing as those around them—and a fear of social sanctioning. This latter channel is precisely what has been emphasized in the empirical literature as a solution to the commitment problem faced by prospective vote buyers, as voters police one another's decision to vote for the "right" candidate (Nichter 2008; Finan and Schechter 2012; Calvo and Murillo 2013), making it particularly pertinent.

An important caveat regarding network dependence is that we normalize the *total* social influence of any voter's connections to 1, so that all voters place equal weight on their neighbors' vote probabilities. Intuitively, this implies that the influence exerted on voter $i$ by their network neighbor $j$ is greater if $j$ is $i$'s *only* neighbor than it would have been if $i$ had been connected to a hundred other voters. Equivalently, voters can be thought of as making their decisions based on a weighted average of their neighbors' actions, and not a sum. While we feel that this accurately captures the actual strategic environment, it is worth noting that this aspect of the model diverges somewhat from others in the literature, such as that of Battaglini and Patacchini (2018), who assume implicitly that the most connected voters are also the most easily influenced.

Finally, candidates care about both their electoral performance their "programmatic" commitment to provide a public good, but face a potential trade-off between these two goals. The funds for private transfers to voters, which can provide an electoral advantage, must be diverted from the provision of the public good. In order to maximize the clarity of the results, we assume that candidates attempt to maximize vote share rather than to win a majority, although simulations indicate that our main results are substantively unchanged if candidates only value winning. In addition, this assumption is empirically appropriate in many cases where vote buying is common, especially in authoritarian regimes, where incumbents frequently seek to achieve overwhelming vote shares, not just a majority (Reuter and Robertson 2012).

## 2.1 Setup

Formally, consider a game with $n$ voters that need to make a choice between two candidates. All players of the game are located on a network $\mathcal{G}$, which is assumed to be completely connected.[3] Let the policy space be a subset of $\mathbb{R}$. Each candidate $k = 1, 2$ is associated with a policy $y_k = k$ and each voter $i \in N := \{1, \ldots, n\}$ is endowed with a group membership $\ell_i = 1, 2$, which corresponds to an ideal policy $x_i = \ell_i$. Substantively, groups may be interpreted as corresponding to any grouping that is both socially and politically meaningful, such as ethnic or religious groups, political parties, or voting blocs.

In attempt to gain vote share, candidate $k$ can extend a private bribe $b_{ik} \geq 0$ to any voter $i$. These bribes, however, come at the expense of a public good, which the

[3] We use the terms *network* and *graph* interchangeably throughout the paper to refer to an undirected graph, which is an ordered pair $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V}$ is a set of $n + 2$ vertices and $\mathcal{E}$ is a set of $m$ edges such that $\mathcal{E} \subseteq \{\{x, x'\} : x, x' \in \mathcal{V} \wedge x \neq x'\}$.

candidate may also care about. A candidate $k$'s problem is to choose $\boldsymbol{b}_k$ that solves

$$\max_{\boldsymbol{b}_k} \;\; \alpha \sum_{i \in N} \phi_{ik}(\boldsymbol{b}_k, \boldsymbol{b}_{1-k}) - \boldsymbol{b}_k \cdot \mathbf{1} \tag{1}$$

$$\text{subject to} \;\; b_{ik} \geq 0 \text{ for all } i \tag{2}$$

where $\phi_{ik}(\cdot)$ is the probability voter $i$ votes for candidate $k$ and 1 dollar in bribes is valued at $\alpha$ in vote probability by the candidate. We thus normalize the value placed on a unit of public good by the candidate to one, so that $\alpha$ can be interpreted as the candidate's relative degree of office motivation. In particular, an $\alpha$ of 0 corresponds to a fully programmatic candidate, who trivially prefers to offer no bribes and promise the full amount of the public good.

Voters support the candidate that offers them a total higher payoff, and have no obligation to vote for a candidate who offered them a bribe. All voters care about policy according to a standard quadratic loss function and have $\gamma \geq 0$ value for a unit of public good, so that they incur a loss of $\gamma$ for every dollar offered by a candidate to *any* voter. Additionally, voters have private information unknown to the candidates and other voters. In particular, voters receive a private valence shock for each candidate, $\varepsilon_{ik}$. Without loss of generality, we can normalize $\varepsilon_{i2} = 0$ and define $\varepsilon_i := \varepsilon_{i1}$, which we assume is an independent, uniformly distributed random variable with mean zero and density $\theta > 0$. We interpret $\theta$ as the candidates' information, with smaller $\theta$ indicating less informed candidates. $\theta$ can also be taken as reflecting the intensity of the commitment problem facing candidates, as candidates with higher values can be more certain that a transfer to a voter will actually secure their vote.

Social structure also matters. In particular, voters prefer to vote for the same candidate as their neighbors. Denote by $\phi_{ik}(\cdot)$ the probability voter $i$ votes for candidate $k$ given all bribes, but before the realization of $\varepsilon_i$. Then, each voter $i$ places weight

$w_{ij} \geq 0$ on voter $j$'s probability of voting for candidate $k$ if $i$ and $j$ are connected, and 0 otherwise. In the realized graph $\mathcal{G}$, the set of a voter $i$'s social ties is denoted by $\mathcal{T}_i(\mathcal{G}) \subseteq N$. The total social influence on each voter is normalized to 1, so that the actual influence of each neighbor $j$ on $i$'s utility is equal to $\left(\sum_{h \in \mathcal{T}_i(\mathcal{G})} w_{ih}\right)^{-1} w_{ij}$, implying that more highly connected voters are less influenced by each individual neighbor.

The expected payoff voter $i$ receives by voting for candidate $k$ can thus be expressed as

$$U_i(k) = -(x_i - y_k)^2 + u(b_{ik}) + \left( \sum_{h \in \mathcal{T}_i(\mathcal{G})} w_{ih} \right)^{-1} \sum_{j \in \mathcal{T}_i(\mathcal{G})} w_{ij} \phi_{jk}(\boldsymbol{b}) - \gamma \sum_{m \in N} b_{mk} + \varepsilon_{ik} \quad (3)$$

where $u(\cdot)$ is voter utility over bribes, which we assume is strictly increasing with diminishing marginal returns and that the rate of diminution is decreasing. That is, we require that the utility over bribes satisfy $u'(\cdot) > 0$, $u''(\cdot) < 0$, $u'''(\cdot) \geq 0$, $\lim_{b \to 0} u'(b) = \infty$, and $\lim_{b \to \infty} u'(b) = 0$. These assumptions include a wide range of plausible utilities, particularly logarithmic utility, and ensure that all solution objects are well-defined, as well as ruling out the combinatoric problem of corner solutions.

### Timing

The timing of the game is as follows.

1. Nature randomly chooses a private net valence shock for each voter, $\varepsilon_i \sim \mathcal{U}\left[\frac{-1}{2\theta}, \frac{1}{2\theta}\right]$
2. For all voters $i \in N$, each candidate $k = 0, 1$ offers a bribe $b_{ik} \geq 0$, which determines the residual public good offered
3. Each voter $i \in N$ votes $v_i \in \{0, 1\}$

## 2.2 Equilibrium

A voter will cast a ballot for candidate 1 if and only if $U_i(1) \geq U_i(2)$. Here, candidates will not be able to perfectly anticipate voting behavior due to their imperfect information over voter preferences. such that we can rewrite this condition as

$$\varepsilon_i \leq (-1)^{x_i} + w(b_{i1}) - w(b_{i2}) + \frac{\sum_{j \in \mathcal{T}_i(\mathcal{G})} w_{ij}(2\phi_j - 1)}{\sum_{j \in \mathcal{T}_i(\mathcal{G})} w_{ij}} + \gamma \sum_{m \in N} (b_{m2} - b_{m1}).$$

Denoting $\phi_i := \phi_{i0}(\boldsymbol{b}) = 1 - \phi_{i1}(\boldsymbol{b})$ the probability a voter $i$ votes for candidate 0 and noting that for $\varepsilon_i \sim \mathcal{U}\left[\frac{-1}{2\theta}, \frac{1}{2\theta}\right]$ implies $\Pr(\varepsilon_i \leq \varepsilon) = \frac{1}{2} + \theta\varepsilon$, we can correspondingly write each voter's probability for voting for candidate 1 as

$$\begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix} = \begin{pmatrix} \frac{1}{2} + \theta\left((-1)^{x_1} + w(b_{11}) - w(b_{12}) + \frac{\sum_{j \in \mathcal{T}_1(\mathcal{G})} w_{1j}(2\phi_j - 1)}{\sum_{j \in \mathcal{T}_1(\mathcal{G})} w_{1j}} + \gamma \sum_{m \in N}(b_{m2} - b_{m1})\right) \\ \vdots \\ \frac{1}{2} + \theta\left((-1)^{x_n} + w(b_{n1}) - w(b_{n2}) + \frac{\sum_{j \in \mathcal{T}_n(\mathcal{G})} w_{nj}(2\phi_j - 1)}{\sum_{j \in \mathcal{T}_n(\mathcal{G})} w_{nj}} + \gamma \sum_{m \in N}(b_{m2} - b_{m1})\right) \end{pmatrix}.$$

Here, $\boldsymbol{\phi}$ gives the unique vector of equilibrium vote probabilities. Note that, while each voter's utility is subject only to their neighbor's vote probabilities, this system of equations necessarily implies that a single voter's probability of supporting candidate 1 is a function of all other voter's probability of supporting 1. This occurs because, for example, a voter $i$'s probability $\phi_i$ is affected by $i$'s neighbor $j$'s probability $\phi_j$, which in turn is affected by $j$'s neighbor $m$'s probability $\phi_m$. Since we rule out disconnected components, $\phi_i$ will both affect and be affected by all other voting probabilities throughout the entire network.

In equilibrium, candidate 1 chooses a vector of bribes that solves $\max_{\boldsymbol{b}_1 \in \mathbb{R}^n_+} \sum_i \phi_i$ taking candidate 2's bribes as given, while candidate 1 chooses a vector of bribes to minimize the same objective function, taking candidate 1's bribes as given. This gives rise to the $n$ first-order conditions,

$$\sum_{j=1}^{n} \frac{\partial \phi_j}{\partial b_{ik}} = \frac{1-\lambda_k}{\alpha}$$

where $\lambda_k$ is the Lagrangian multiplier associated with $k$'s non-negativity constraint. Note that differentiating $\phi_i$ with respect to a bribe from candidate 1 to another voter $h$, we have

$$\frac{\partial \phi_i}{\partial b_{h1}} = \theta \left( u'(b_{h1})\mathbb{1}(i=h) + \frac{2\sum_{j\in\mathcal{T}_i(\mathcal{G})} w_{ij}\phi_j'}{\sum_{j\in\mathcal{T}_i(\mathcal{G})} w_{ij}} - \gamma \right) \tag{4}$$

Using this and the first-order conditions, we can rewrite the candidates' problem as

$$(\boldsymbol{J}[\boldsymbol{u}] - \boldsymbol{\Gamma})^{\top} \cdot \left(\boldsymbol{I} - 2\theta\tilde{\boldsymbol{A}}\right)^{-1} \cdot \mathbf{1} = \frac{(1-\boldsymbol{\lambda})}{\alpha\theta} \tag{5}$$

where $\boldsymbol{J}[\cdot]$ is a diagonal matrix with $u'(b_i)$ as the non-zero entries, $\boldsymbol{\Gamma}$ is an $n \times n$ square matrix such that every element of $\boldsymbol{\Gamma}$ is $\gamma$, $\boldsymbol{I}$ denotes the identity matrix, $\mathbf{1}$ denotes an $n$-vector of 1s, $\boldsymbol{\lambda}$ is an $n$-vector of Lagrange multipliers, and $\tilde{\boldsymbol{A}}$ is a normalized weighted adjacency matrix.

**Definition 1.** *Consider a realized graph $\mathcal{G}$ and a corresponding adjacency matrix $\boldsymbol{A}$ such that for all $i, j \in N$, $A_{ij} = 1$ if $j \in \mathcal{T}_i(\mathcal{G})$ and $A_{ij} = 0$ otherwise. Then, the* ***normalized weighted adjacency matrix*** $\tilde{\boldsymbol{A}}$ *is given by, for all $i, j \in N$,*

$$\tilde{A}_{ij} = \frac{w_{ij}A_{ij}}{\sum_{m\in N} w_{im}A_{im}}.$$

By employing the normalized weighted adjacency matrix, we can account for several important features of social interaction. First, a voter $i$ may be more influenced by one social tie than another. Second, it will be more difficult to influence a highly connected voter than a relatively disconnected one, e.g. an incremental change in the probability that $i$'s neighbor $j$ votes for candidate 0 will have less of an effect on $i$'s vote probability if $i$ has hundreds of neighbors than if $j$ is $i$'s only neighbor.

From the candidates problem in equation (5), we can recover the equilibrium bribe from candidate $k$ received by voter $i$,

$$b_{ik} = [u']^{-1} \left( \gamma n + \frac{1}{\alpha \theta c_i} \right) \tag{6}$$

where $c_i$ is the $i$th element of $\boldsymbol{c}$ and $\boldsymbol{c} = (\boldsymbol{I} - \theta \tilde{\boldsymbol{A}})^{-1}\mathbf{1}$ is our measure of centrality. This measure corresponds to each agent's Katz-Bonacich centrality on the weighted directed network corresponding to $\tilde{\boldsymbol{A}}$ with attenuation parameter $\theta$. The nature of the strategic environment—specifically, the structure of social influence—can thus be thought of as inducing a latent directed network with connections corresponding to the influence of $i$ on $j$, which is decreasing in $j$'s weighted degree and highest when $\ell_i = \ell_j$. The value of a voter to a candidate is thus proportional to their centrality on this latent network, which captures the weighted sum of directed walks of any length that include that voter.

As the characterization makes clear, $b_{ik}$ does not rely on policy considerations and hence, in equilibrium, both candidates bribe a given voter the same amount according to their centrality on the network.

**Proposition 1** (Equality of bribes)**.** *In any equilibrium,* $b_{i1} = b_{i2}$ *for all* $i \in N$*.*

To understand the intuition behind this result, recall that a voter's direct utility in bribes is increasing with diminishing marginal returns. Then, either candidate $k$ will want to continue extending bribes to a voter $i$ until the marginal gain in vote probability is equal to the marginal loss is public good. Since candidates want to maximize their total expected vote share, the point at which this occurs for voter $i$ is the same for candidate 1 as it is for candidate 2. Voters will not receive more private inducements from candidates they are politically aligned with than from those they are not. Proposition 1 implies that the net direct gain from bribes is always zero, i.e. $u(b_{i1}) - u(b_{i2}) = 0$. Note also that $\gamma \sum_{i \in N}(b_{i2} - b_{i1}) = 0$. Together, these facts lead to the next result.

**Proposition 2** (Electoral outcomes)**.** *Vote buying does not affect electoral outcomes. In particular, the probability a voter $i$ votes for candidate 1 is*

$$\phi_i = \frac{1}{2} + \theta \left( (-1)^{x_i} + \sum_{j \in \mathcal{T}_i(\mathcal{G})} 2w_{ij}(\phi_j - 1) \right)$$

*for all $i \in N$, which is independent of candidate strategies.*

Since an individual voter receives the same bribe from each candidate, neither candidate is successful in affecting change in the voter's probability of supporting them in the election. As no voter is influenced by the bribes (or, arguably, as all voters are influenced by the bribes equally in both directions), the equilibrium expected vote share is not altered by the candidates' bribes. Importantly, however, this is a feature of the two-candidate competitive environment. If, for instance, candidate 2 lacks the resources to distribute bribes or is ideologically committed to programmatic politics, then candidate 1 could achieve a large expected vote share, even if a majority of voters prefer candidate 2 on ideological grounds.

Finally, it is straightforward from the assumptions on the derivatives of $u(\cdot)$ stated in the previous section to derive the following comparative statics by taking derivatives of the equilibrium bribes defined by equation 6.

**Proposition 3** (Comparative Statics)**.** *For any voter $i$, the equilibrium transfers offered by both candidates are*

1. *Weakly decreasing in $\gamma$*

2. *Weakly decreasing in $n$*

3. *Weakly increasing in $\alpha$*

4. *Weakly increasing in $\theta$*

These results are consistent with the intuition of the basic strategic environment: the socially optimal bribes to voters would correspond to a transfer scheme such that the marginal value is equated with $\gamma$, but candidates provide additional transfers as a result of the network spillovers. The value of these spillovers is moderated by $\alpha$—that is, more office-motivated candidates place higher value on the additional increase to expected vote share—and by $\theta$, which determines the likelihood of their realization.

Thus far, we have taken the exact realized network as given. In order to study the dependence of equilibrium strategy on social structure, we must transition to considering the underlying generating model that gave rise to the observed network. We now take up this problem in the next section.

# 3 Analysis

This section proceeds as follows. First, we present results related to graph theory that justify analysis of an average graph. This is needed for two reasons: since centrality is a highly nonlinear function of the realized network, it is convenient to replace the expected centrality of each individual voter—which is defined only implicitly—with that voter's centrality *on the expected network*, which is far more tractable. In addition, even simple generative models may result in realized networks with arbitrarily high variance in the emergent characteristics of individual vertices, such as centrality. We therefore require bounds on the spectrum of the induced normalized weighted network in order facilitate making claims that can be made to hold with arbitrarily high probability. These results are technical in nature and therefore detailed proofs are relegated to the Appendix.

Second, we employ these tools to study the implications of social structure on vote buying. In particular, we are able to derive closed-form expressions for the centrality of voters in each party, yielding sharp comparative statics in terms of the main features of social structure: namely, group fractionalization and homophily.

## 3.1 Graph Theory Results

The main result, which is an analogue of the spectral theorems of Chung and Radcliffe (2011) applied to the case of weighted, directed networks, allows us to place tight bounds on the deviation of the realized normalized weighted adjacency matrix from its expected counterpart.

**Theorem 1.** *Let $\boldsymbol{L}_W$ denote the normalized weighted Laplacian of $\mathcal{G}$, $\omega$ the largest total weight, and $\underline{d}$ the smallest expected degree. For any $\epsilon > 0$, there exists a $k(\epsilon)$ such that, for all $i$,*

$$\Pr\left( ||\boldsymbol{L}_W - \bar{\boldsymbol{L}}_W|| \leq 4\sqrt{\frac{3\omega \ln(4n/\epsilon)}{\underline{d}}} \right) \geq 1 - \epsilon$$

*if $\underline{d} > k\ln(n)$ and $\alpha\omega \leq \sqrt{\frac{\underline{d}}{3\ln(4n/\epsilon)}}$, where $\alpha$ is the smallest total weight.*[4]

A major limitation of this result, however, is that the bound depends on $n$, and is thus arbitrary large in large societies for a fixed $\epsilon$. Under the assumption that the minimum degree grows at a rate greater than $\ln(n)$,[5] it is straightforward to show the following result, which allows us to make asymptotic statements with high probability.

[4] Not to be confused with $\alpha$ in candidate utility.

[5] In fact, empirical work suggests that it grows at approximately rate $\ln(n)$ in most societies. However, simulation studies indicate that our results still hold with high probability if this is the case, and it is likely that a tighter bound is achievable.

**Lemma 1.** *For any* $\epsilon > 0$*,* $\lim_{n\to\infty} \Pr(\|\boldsymbol{c}^{(n)}(\tilde{\boldsymbol{A}}) - \boldsymbol{c}^{(n)}(\bar{\tilde{\boldsymbol{A}}})\| > \epsilon) = 0$*.*

This therefore allows us to consider centrality on the average graph only, permitting analysis of comparative statics in terms of social structure, rather than of a single realized graph.

## 3.2 Social Structure

**Definition 2.** *The* ***average normalized weighted adjacency matrix*** $\bar{\tilde{\boldsymbol{A}}}$ *is given by, for all* $i, j \in N$*,*

$$\bar{\tilde{A}}_{ij} = \frac{w_{ij}p_{ij}}{\sum_{m\in N} w_{im}p_{im}}.$$

For the following results, we assume that the graph is drawn according to a two-group stochastic block model with share $s \geq \frac{1}{2}$ of group 1,[6] a probability $p_H$ of intra-group connection, and a probability $p_L$ of inter-group connection. Further, assume for simplicity that $w_{ij} = w_H$ for in-group voters and $w_{ij} = w_L$ for out-group voters, with the natural assumption that $w_H \geq w_L$. Finally, we denote by $0 < \delta < 1$ the ratio $\frac{w_L p_L}{w_H p_H}$, which thus captures the degree of homophily on the network (lower delta corresponds to more homophily).

The main result, which draws on the asymptotic bounds on the average adjacency matrix derived in the previous section, allows us to derive closed-form expressions for each voter's centrality that hold with high probability given large $n$.

**Proposition 4** (Expected Centrality)**.** *For* $n$ *sufficiently large, with probability approaching one the centrality of a voter in party* 1 *is*

$$c_1 = \frac{-\delta + (\delta - 1)s^2(\delta\theta + \delta + \theta - 1) - (\delta - 1)s(\delta\theta + \delta + \theta - 1)}{sn((\delta - 1)s - \delta)(-\theta + s(\delta + \theta - 1) + 1)} \tag{7}$$

[6] Notably, in the two-group case this parameter captures all of the information provided by the Herfindahl-Hirschman index, which is widely used as a measure of ethnolinguistic fractionalization in empirical research.

*and for a voter in party 2,*

$$c_2 = \frac{\delta - ((\delta-1)s^2(\delta\theta+\delta+\theta-1)) + (\delta-1)s(\delta\theta+\delta+\theta-1)}{n(s-1)((\delta-1)s+1)(s(\delta+\theta-1)-\delta)} \tag{8}$$

The proof of this result is left to the Appendix, but the key observation is that unlike *realized* networks, which may be arbitrarily complex, the *expected* network is necessarily complete, since all voters have positive probability of being connected to all others. In particular, the adjacency matrix, while arbitrarily large, contains only four unique values, corresponding to directed connections within and between each group. With some algebra, it is therefore possible to derive an explicit formula for the inverse of this matrix, which in turn determines the value of each voter's centrality.

An immediate and remarkable conclusion that follows from Proposition 4 is that while centrality is a function of group sizes,[7] information, and homophily, it is entirely *independent* of the actual density of the network. Put differently, a uniform increase in connection probabilities between all voters would not influence equilibrium bribes in any way. Despite the prominence of density in many informal accounts of network effects, then, our model suggests this need not be a major determinant of the efficacy of vote buying, further highlighting the need for precisely formulated theory.

Also of note is that, under the assumption that $s \geq \frac{1}{2}$, it can easily be verified that $c_1 \leq c_2$, with equality only if the groups are of exactly equal size. In other words, each individual member of the minority group will always receive a higher equilibrium transfer than a member of the majority. Intuitively, this is driven by the fact that, assuming any positive degree of homophily, the influence of each member of the minority group increases as the group becomes smaller, making them more valuable to target.

Since these centralities are smoothly differentiable functions of all parameters,

[7] Note that while the centrality of each individual voter is decreasing in $n$, the total centrality is constant.

moreover, it is then straightforward to analyse how the total spending of candidates, as well as the level of between-group inequality, depends on these parameters. In particular, let $B$ denote the sum of bribes across all voters and $Q$ denote the level of inequality, or $Q = |1 - b_1/b_2|$. Since $u(\cdot)$ is assumed to be concave with convex first derivative, it is sufficient to consider the total centrality rather than examining bribes directly. Then we have the following result.

**Proposition 5** (Total bribes)**.** *For all* $\theta \leq 2 - \sqrt{2}$*,* $\frac{\partial B}{\partial \delta} > 0$*,* $\frac{\partial B}{\partial s} > 0$*, and* $\frac{\partial B}{\partial \theta} > 0$*.*

Since $\theta$ needs to be small for the solution to be well-defined, the condition for these results to hold will always hold in practice for large $n$. The effect of $\theta$ here is intuitive: better information increases the expected return on bribes. The other two results are non-obvious, however. In particular, the effect of $\delta$ is quite counter-intuitive: an increase in $\delta$, which corresponds to a *decrease* in homophily, actually increases total spending. While the mechanism for this is straightforward—higher $\delta$ corresponds to stronger "weak ties", raising the value of transfers to all voters—it is again at odds with many informal descriptions of "dense ethnic networks" that are argued to be particularly amenable to clientelism because of their high degree of homophily.

It is similarly straightforward to study the effect of network parameters on inequality by taking derivatives of the negative of the ratio of the two centralities:

**Proposition 6** (Group size and inequality)**.** *Let* $Q$ *denote the total inequality, or* $Q = |1 - b_1/b_2|$*. Then* $\frac{\partial Q}{\partial \theta} \geq 0$*, with equality if* $s = \frac{1}{2}$ *or* $s = 1$*, and* $\frac{\partial Q}{\partial s} > 0$ *for all parameter values. Moreover,* $\frac{\partial Q}{\partial \delta} < 0$ *only if* $\delta \geq \sqrt{1 - \theta}$*, and is positive otherwise.*

Once again, the effects of information and group size are intuitive, suggesting that better informed candidates in more demographically uneven societies will concentrate their resources more intensely in the groups that provide the highest return. As with total spending, however, the effects of homophily are surprising. While higher homophily (lower $\delta$) can increase inequality, this only holds for networks that exhibit extremely low degrees of homophily. In contrast, at all empirically plausible levels of homophily, increasing the relative influence of voters on members of their own group actually decreases the overall inequality of transfers from candidates. This result is particularly remarkable given that Dasaratha (2020) arrives at the opposite conclusion regarding Katz centrality on an undirected and unweighted network.

In fact, the key to understanding this result is the tradeoff faced by candidates between targeting highly-connected voters, who influence many others, and voters whose neighbors are *not* highly connected, since they are more easily influenced. In the extreme, as $\delta$ approaches 0, the greater value of transfers to members of the minority is completely offset by their disconnectedness from the majority, such that the equilibrium bribes approach equality.

# 4 Discussion

In this article, we provide a formal model of vote buying on a network of policy-motivated voters who care about both private and public goods and candidates who can extend bribes to improve their electoral performance. We study the game under a given network structure and show that, in equilibrium, each individual voter receives the same bribe from both candidates. Since no candidate receives more bribes from one candidate than the other, equilibrium expected vote share remains unchanged and hence bribery does not affect the electoral outcomes.

Additionally, we employ techniques from advances in spectral random graph theory to research the role of social structure. In doing so, we are able to explicitly characterize the equilibrium bribes received by each voter in large networks and thus to derive sharp comparative statics regarding the role of social structure. Particularly noteworthy are our findings regarding density and homophily. Contrary to arguments frequently found in the literature, density does not affect either the level of group inequality or total spending, while homophily actually decreases both for most plausible parameter combinations.

These results have direct implications on vote buying in a variety of political settings. For instance, we predict the most intense vote-buying to occur in social contexts with a large majority and small minority but with relatively low levels of social segregation. At the same time, members of the minority are likely to benefit disproportionately from vote-buying, especially when candidates are well-informed about voter preferences.

There are a number of interesting questions our model provokes that we leave to future research. For example, the voters in our model do not strategically transmit information to their neighbors, but instead transparently share their vote probabilities. A natural extension is therefore to allow voters to attempt to persuade their neighbors by not fully revealing their truth with the goal of maximizing the transfers they receive. Such a model may be able to answer questions about how polarization in a society affects vote buying and how large, heterogeneous biases may interact with features of a social network studied in this paper, such as density and homophily.

Another aspect of vote buying that would be valuable to incorporate in the existing model is the role of social connections between the candidate and voters. In particular, consider a related model in which candidates have limited information about the network structure, but can pay a cost that is proportional to their social distance to discover pertinent information about voters, such as party membership and policy preferences. This extension could inform how candidate location in the existing social structure affects the extent and distribution of bribery, as well as accounting for the empirically observed tendency to favor members of the candidate's ingroup.

# Appendix

**Lemma 2** (Chung and Radcliffe (2011)). *Let* $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_m$ *be bounded independent random Hermitian matrices and set* $M > 0 : ||\boldsymbol{X}_i - E(\boldsymbol{X}_i)||_2 \leq M \; \forall i = 1, \ldots, m$. *Then for any* $a > 0$,

$$\Pr(||\boldsymbol{X} - E(\boldsymbol{X})||_2 > a) \leq 2n \exp\left(-\frac{a^2}{2v^2 + 2Ma/3}\right)$$

*where* $\boldsymbol{X} = \sum_{i=1}^{m} \boldsymbol{X}_i$ *and* $v^2 = ||\sum_{i=1}^{m} \mathbb{V}(\boldsymbol{X}_i)||$.[8]

*Proof of Theorem 1.* Let $\mathcal{G}$ be an undirected random graph such that all edge formation probabilities are jointly independent. Denote by $\boldsymbol{A}$ the adjacency matrix, $\boldsymbol{W}$ a matrix of weights, and $\boldsymbol{A}_W$ the weighted adjacency matrix, such that $\boldsymbol{A}_W = \boldsymbol{W} \odot \boldsymbol{A}$.[9] Let $\boldsymbol{D}_W$ be the diagonal degree matrix such that $\{\bar{\boldsymbol{D}}_W\}_{ii} = \sum_j w_{ij} a_{ij}$, and denote by $\bar{\boldsymbol{A}}, \bar{\boldsymbol{D}}_W$ the expected equivalents. Finally, let $\boldsymbol{L}_W = \boldsymbol{I} - \boldsymbol{D}_W^{-1/2} \boldsymbol{A}_W \boldsymbol{D}_W^{-1/2}$ denote the normalized weighted Laplacian of $\mathcal{G}$, $\omega = \max_{i,j} w_{ij}$ be the largest total weight, $\alpha = \min_{i,j} w_{ij}$ the smallest, and $\delta = \min_i \{\bar{\boldsymbol{D}}_W\}_{ii}$ the smallest expected degree.

Denote $\bar{d}_i$ as the expected (weighted) degree of node $i$. By the triangle inequality, for any matrix $C$,

$$||\boldsymbol{L}_W - \bar{\boldsymbol{L}}_W|| \leq ||\boldsymbol{C} - \bar{\boldsymbol{L}}_W|| + ||\boldsymbol{L}_W - \boldsymbol{C}||$$

In particular, let $\boldsymbol{C} = I - \bar{\boldsymbol{D}}_W^{-1/2} \boldsymbol{A}_W \bar{\boldsymbol{D}}_W^{-1/2}$. Then since the degree matrices are diagonal, we have $\boldsymbol{C} - \bar{\boldsymbol{L}}_W = \bar{\boldsymbol{D}}_W^{-1/2} (\boldsymbol{A}_W - \bar{\boldsymbol{A}}_W) \bar{\boldsymbol{D}}_W^{-1/2}$. Denoting by $\boldsymbol{A}^{ij}$ the matrix that is equal to 1 in the $i, j$th and $j, i$th positions and 0 elsewhere, we can write $i, j$th

[8] See Theorem 5 in Chung and Radcliffe (2011) for the proof.
[9] Note that $\odot$ indicates the Hadamard (element-wise) product.

entry of $\boldsymbol{C} - \bar{\boldsymbol{L}}$ as

$$\boldsymbol{X}_{ij} = \bar{\boldsymbol{D}}_W^{-1/2}(w_{ij}(a_{ij} - p_{ij})\boldsymbol{A}^{ij})\bar{\boldsymbol{D}}_W^{-1/2} = \frac{w_{ij}(a_{ij} - p_{ij})}{\sqrt{\bar{d}_i \bar{d}_j}}\boldsymbol{A}^{ij}$$

Then clearly $\boldsymbol{C} - \bar{\boldsymbol{L}} = \sum \boldsymbol{X}_{ij}$, so Lemma 2 applies. Note that since $E(a_{ij}) = p_{ij}$, we have that $E(\boldsymbol{X}_{ij}) = \mathbf{0}$, so that $v^2 = ||\sum E(\boldsymbol{X}_{ij}^2)||$. Also, each $\boldsymbol{X}_{ij}$ is bounded above by $\|\boldsymbol{X}_{ij}\| \leq \frac{\omega}{\delta}$ Now clearly

$$E(\boldsymbol{X}_{ij}^2) = \begin{cases} \frac{w_{ij}^2}{\bar{d}_i \bar{d}_j}(p_{ij})(1 - p_{ij})(A^{ii} + A^{jj}) & i \neq j \\ \frac{w_{ii}^2}{\bar{d}_i^2}(p_{ij})(1 - p_{ij})A^{ii} & i = j \end{cases}$$

Now we can write

$$\begin{aligned} v^2 &= \left\| \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{w_{ij}^2}{\bar{d}_i \bar{d}_j}(p_{ij})(1 - p_{ij})A^{ii} \right\| \\ &= \max_i \left( \sum_{j=1}^{n} \frac{w_{ij}^2}{\bar{d}_i \bar{d}_j}(p_{ij})(1 - p_{ij}) \right) \\ &\leq \max_i \left( \frac{\omega}{\delta} \sum_{j=1}^{n} \frac{w_{ij}}{\bar{d}_i}(p_{ij}) \right) \\ &= \frac{\omega}{\delta} \end{aligned}$$

For notational convenience denote $a = \sqrt{\frac{3\omega \ln(4n/\epsilon)}{\delta}}$ and $\delta$ so that $a < 1$. In particular, we must have $\delta > 3\omega(\ln(4) + \ln(n) - \ln(\epsilon))$, so that if $k \geq 3\omega(1 + \ln(4/\epsilon))$, $\delta \geq k \ln(n)$

guarantees the result. Then from Lemma 2

$$\begin{aligned}
\Pr(\|\boldsymbol{C} - \bar{\boldsymbol{L}}_W\| > a) &\leq 2n \exp\left(-\frac{\frac{3n\omega^2 \ln(4n/\epsilon)}{n\delta}}{2n\omega^2/\delta + 2an\omega^2/3\delta}\right) \\
&= 2n \exp\left(-\frac{\frac{3n\omega^2 \ln(4n/\epsilon)}{\delta}}{2n\omega^2(3+a)/3\delta}\right) \\
&= 2n \exp\left(-\frac{9\ln(4n/\epsilon)}{6+2a}\right) \\
&\leq 2n \exp\left(-\frac{9\ln(4n/\epsilon)}{9}\right) \\
&= \frac{\epsilon}{2}
\end{aligned}$$

Now for the second term, note that $d_i$ is a sum of random variables that are bounded between 0 and $\omega$. Then by Hoeffding's Inequality, we have that, for any $t$,

$$\Pr(|d_i - \bar{d}_i| > t\bar{d}_i) \leq 2\exp\left(-\frac{t^2\bar{d}_i^2}{n\omega^2}\right) \leq 2\exp\left(-\frac{t^2\delta^2}{n\omega^2}\right)$$

Now in particular let $t = \sqrt{\frac{n\omega^2 \ln(4n/\epsilon)}{\delta^2}} = \sqrt{\frac{n\omega}{3\delta}}a$. We have $t < a < 1$ if $\delta > \frac{n\omega}{3}$. In our application, $\omega = \rho_H, \delta = n_0 p_L \rho_L + n_1 p_H \rho_H.$, so that for all $i$ we obtain

$$\Pr(|d_i - \bar{d}_i| > t\bar{d}_i) \leq \frac{\epsilon}{2n}$$

Now note that

$$\left\|\bar{\boldsymbol{D}}_W^{-1/2}\boldsymbol{D}_W^{1/2} - \boldsymbol{I}\right\|_2 = \max_i \left|\sqrt{\frac{d_i}{\bar{d}_i}} - 1\right|$$

To bound this, note that from (4) we can conclude that $\Pr\left(\left|\frac{d_i}{\bar{d}_i} - 1\right| > t\right) \leq \frac{\epsilon}{2n}$ and hence with probability at least $1 - \frac{\epsilon}{2n}$,

$$\left\|\bar{\boldsymbol{D}}_W^{-1/2}\boldsymbol{D}_W^{1/2} - \boldsymbol{I}\right\|_2 < \sqrt{\frac{n\omega^2 \ln(4n/\epsilon)}{\delta^2}}$$

Finally, note that since $\|\boldsymbol{L}\|_2 \leq 2$ (Chung and Graham 1997), we have $\|\boldsymbol{I}-\boldsymbol{L}\|_2 \leq 1$. Now consider

$$\begin{aligned}
\|\boldsymbol{L}_W - \boldsymbol{C}\| &= \|\boldsymbol{I} - \boldsymbol{D}_W^{-1/2}\boldsymbol{A}_W\boldsymbol{D}_W^{-1/2} - I + \bar{\boldsymbol{D}}_W^{-1/2}\boldsymbol{A}_W\bar{\boldsymbol{D}}_W^{-1/2}\| \\
&= \|(\boldsymbol{I} - \boldsymbol{L}_W)\bar{\boldsymbol{D}}_W^{-1/2}\boldsymbol{D}_W^{1/2}\boldsymbol{D}_W^{-1/2}\boldsymbol{A}_W\boldsymbol{D}_W^{-1/2}\boldsymbol{D}_W^{1/2}\bar{\boldsymbol{D}}_W^{-1/2}\| \\
&= \|(\boldsymbol{I} - \boldsymbol{L}_W)\bar{\boldsymbol{D}}_W^{-1/2}\boldsymbol{D}_W^{1/2}(\boldsymbol{I} - \boldsymbol{L})\boldsymbol{D}_W^{1/2}\bar{\boldsymbol{D}}_W^{-1/2}\| \\
&= \|(\bar{\boldsymbol{D}}_W^{-1/2}\boldsymbol{D}_W^{1/2} - \boldsymbol{I})(\boldsymbol{I} - \boldsymbol{L}_W)\boldsymbol{D}_W^{1/2}\bar{\boldsymbol{D}}_W^{-1/2} + (\boldsymbol{I} - \boldsymbol{L})(\boldsymbol{I} - \boldsymbol{D}_W^{1/2}\bar{\boldsymbol{D}}_W^{-1/2})\| \\
&\leq \|\bar{\boldsymbol{D}}_W^{-1/2}\boldsymbol{D}_W^{1/2} - \boldsymbol{I}\|\|\boldsymbol{D}_W^{1/2}\bar{\boldsymbol{D}}_W^{-1/2}\| + \|\boldsymbol{I} - \boldsymbol{D}_W^{1/2}\bar{\boldsymbol{D}}_W^{-1/2}\| \\
&\leq t^2 + 2t
\end{aligned}$$

Hence, finally,

$$\begin{aligned}
||\boldsymbol{L}_W - \bar{\boldsymbol{L}}_W|| &\leq ||\boldsymbol{C} - \bar{\boldsymbol{L}}_W|| + ||\boldsymbol{L}_W - \boldsymbol{C}|| \\
&\leq a + \frac{n\omega}{3\delta}a^2 + \sqrt{\frac{4n\omega}{3\delta}}a \\
&= a\left(\frac{\sqrt{3\delta} + 2\sqrt{n\omega}}{\sqrt{3\delta}} + \frac{n\omega}{3\delta}a\right) \\
&= a\left(1 + \frac{2\sqrt{3n\omega\delta} + n\omega a}{3\delta}\right)
\end{aligned}$$

Now, choose $k > 1$ such that

$$\delta \geq \frac{1}{3}\left(2n\omega\frac{\sqrt{k} + k + 1}{(k-1)^2}\right)$$

□

*Proof of Proposition 4.* By the result established in Lemma 1, it is sufficient to consider centrality on the average network. Under the stochastic block model, letting $s_i$

denote the share of $i$'s group without loss of generality, we have that the expected degree of $i$ can be written as

$$\sum_{j=1}^{n} w_{ij}p_{ij} = s_i n w_H p_H + (1-s_i) n w_L p_L = n\left(w_L p_L + s_i(w_H p_H - w_L p^{\boldsymbol{L})}\right), ^{10}$$

For notational convenience, we denote $w_H p_H = \rho, w_L p_L = \delta\rho$ for some $0 < \delta < 1$. The key observation is that the actual value of $\rho$ is irrelevant, since it appears in both the denominator and numerator of each entry of the expected adjacency matrix. Thus, all results depend only on $\delta$, the *relative* expected weight placed on out-group connections.

Note now that we can write the matrix $\boldsymbol{I} - \theta\bar{\bar{\boldsymbol{A}}}$ as a $2 \times 2$ block matrix with blocks $\bar{\bar{\boldsymbol{A}}}_{11} = \boldsymbol{I} - \frac{\theta}{n(\delta+s_1(1-\delta))}(\mathbf{1}_{s_1n\times s_1n} - \boldsymbol{I})$, $\bar{\bar{\boldsymbol{A}}}_{12} = -\frac{\theta\delta}{n(\delta+s_1(1-\delta))}\mathbf{1}_{s_1n\times s_2n}$, $\bar{\bar{\boldsymbol{A}}}_{21} = -\frac{\theta\delta}{n(\delta+s_2(1-\delta))}\mathbf{1}_{s_2n\times s_1n}$, and $\bar{\bar{\boldsymbol{A}}}_{22} = \boldsymbol{I} - \frac{\theta}{n(\delta+s_2(1-\delta))}(\mathbf{1}_{s_2n\times s_2n} - \boldsymbol{I})$. To apply the formula for block inversion, we first want to identify $\bar{\bar{\boldsymbol{A}}}_{11}^{-1}$. We conjecture that

$$P = \bar{\bar{\boldsymbol{A}}}_{11}^{-1} = \begin{bmatrix} a_1 & b_1 & \cdots & b \\ b & a & \cdots & b \\ \vdots & \cdots & \ddots & \vdots \\ b & \cdots & \cdots & a \end{bmatrix}$$

Then we have that

$$\begin{bmatrix} 1 & -(n_1-1)\frac{\theta}{n(\delta+s_1(1-\delta))} \\ -\frac{\theta}{n(\delta+s_1(1-\delta))} & \left(1-(n_1-2)\frac{\theta}{n(\delta+s_1(1-\delta))}\right) \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

which indeed has a unique solution. The inverse of the bottom-right block is identical, swapping group indices. Hence, we can construct the centrality vector according to

[10] Technically this is an approximation since $p_{ii} = 0$, but the loss is insignificant for large $n$, which is assumed here.

the formula:

$$\boldsymbol{c} = \begin{bmatrix} \left(\bar{\bar{\boldsymbol{A}}}_{11} - \bar{\bar{\boldsymbol{A}}}_{12}\bar{\bar{\boldsymbol{A}}}_{22}^{-1}\bar{\bar{\boldsymbol{A}}}_{21}\right)^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \left(\bar{\bar{\boldsymbol{A}}}_{22} - \bar{\bar{\boldsymbol{A}}}_{21}\bar{\bar{\boldsymbol{A}}}_{11}^{-1}\bar{\bar{\boldsymbol{A}}}_{12}\right)^{-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{I} & -\bar{\bar{\boldsymbol{A}}}_{12}\bar{\bar{\boldsymbol{A}}}_{22}^{-1} \\ -\bar{\bar{\boldsymbol{A}}}_{21}\bar{\bar{\boldsymbol{A}}}_{11}^{-1} & \boldsymbol{I} \end{bmatrix} \tag{A1}$$

The main-diagonal blocks in the first matrix again have the same structure, with a single value on the main diagonal and another value on the off-diagonal. This has a similar structure to the previous matrix, and the inverse can thus be calculated analogously by solving for main and off-diagonal elements $a_i', b_i'$[11].

*Remark.* There is a substantive interpretation of $a_i'$ and $b_i'$: $a_i'$ is the weighted average of the number of paths back to a voter in party $i$ through the network, while $b_i'$ is the weighted average of the number of paths to someone else in your party through the network. Because in the expected network, all voters are connected to all others, $i$'s centrality does not depend on paths to the other party, because a linear dependence is induced (all paths within party essentially correspond to an equivalent cross-party path).

Substituting these values into Equation A1, we then have that

$$c_i = (1 + n_{-i})a_i' + (1 + n_{-i})(n_i - 1)b_i'$$

or, without loss of generality,

$$\mathrm{c}_1 = \frac{n\left(-\delta\theta - \delta n + (\delta-1)ns^2(\delta\theta+\delta+\theta-1) - (\delta-1)ns(\delta\theta+\delta+\theta-1) + (\delta-1)\theta s\right)}{-\theta^2 + n^3 s((\delta-1)s-\delta)(-\theta+s(\delta+\theta-1)+1) + \theta n^2 s(-\delta-\theta+s(2\delta+\theta-2)+1) + \theta n(\theta+\delta^2\theta s - \delta s + s - 1)}$$

Since $n$ is by assumption large, however, this expression is asymptotically equivalent to its leading term, and we can thus simplify further, writing

$$c_1 \sim \frac{-\delta + (\delta-1)s^2(\delta\theta+\delta+\theta-1) - (\delta-1)s(\delta\theta+\delta+\theta-1)}{sn((\delta-1)s-\delta)(-\theta+s(\delta+\theta-1)+1)} \tag{A2}$$

[11] A unique solution again exists, but we suppress the exact expression as it is extremely complex. The mathematica file used to calculate these values is available upon request from the authors.

$\square$

*Proof of Lemma 1.* Let $\mathcal{G}^{(n)}$ be a sequence of random graphs over $n$ vertices, and denote by $\delta_{(n)}$ the smallest expected weighted degree, i.e. $\delta_{(n)} = \min_i \sum_j w_{ij}^{(n)} p_{ij}^{(n)}$. Further, let $\bar{w}_{(n)} = \max_{i,j} w_{ij}^{(n)}$ and $\underline{w}_{(n)} = \min_{i,j} w_{ij}^{(n)}$ be the largest and smallest individual weights, satisfying $\frac{\bar{w}_{(n)}}{\underline{w}_{(n)}} \leq \omega$ for some $\omega > 0$ for all $n$. Then if there exists a non-decreasing sequence of $k_{(n)} > 0$ such that $\delta_{(n)} \geq k_{(n)} \ln(n)$ and $\underline{w}_{(n)} \cdot \bar{w}_{(n)} = o\left(\sqrt{\frac{\delta_{(n)}}{\ln(n)}}\right)$, then the realized centrality vector centrality vector $\boldsymbol{c}^{(n)}(\tilde{\boldsymbol{A}})$ is with high probability close to the centrality of the average graph $\boldsymbol{c}^{(n)}(\bar{\tilde{\boldsymbol{A}}})$ for large $n$.

Under the stated assumptions, we can apply Theorem 1 to conclude that for any $\xi > 0$, for all $n$ we have

$$\Pr\left(||\boldsymbol{L}_W - \bar{\boldsymbol{L}}_W|| \leq 4\sqrt{\frac{3\omega \ln(4n/\xi)}{\delta}}\right) \geq 1 - \xi$$

Furthermore, by assumption $\lim_{n\to\infty} 4\sqrt{\frac{3\omega \ln(4n/\xi)}{\delta}} = 0$ regardless of the $\xi$ chosen, so that under the 2-norm,

$$\boldsymbol{L}_W \underset{p}{\rightarrow} \bar{\boldsymbol{L}}_W$$

Now, for convenience call $B = \boldsymbol{I} - \boldsymbol{L}_W$ and $\bar{B}$ the expected equivalent. Now clearly also $B \underset{p}{\rightarrow} \bar{B}$, and furthermore we can write $B = \boldsymbol{D}^{-1/2}\boldsymbol{A}_W\boldsymbol{D}^{-1/2} = \boldsymbol{D}^{-1/2}\tilde{A}\boldsymbol{D}^{1/2}$. So we can write, using properties of matrix norms (abusing notation in the second step slightly so that the maximum is over the norm of the matrices) and the above result,

$$\begin{aligned}\limsup_{n\to\infty}\|\tilde{\boldsymbol{A}}-\bar{\tilde{\boldsymbol{A}}}\| &= \limsup_{n\to\infty}\|\boldsymbol{D}^{1/2}\boldsymbol{B}\boldsymbol{D}^{-1/2}-\bar{\boldsymbol{D}}^{1/2}\bar{\boldsymbol{B}}\bar{\boldsymbol{D}}^{-1/2}\| \\ &\leq \limsup_{n\to\infty}\|\max\left\{\boldsymbol{D}^{1/2},\bar{\boldsymbol{D}}^{1/2}\right\}(\boldsymbol{B}-\bar{\boldsymbol{B}})\max\left\{\boldsymbol{D}^{-1/2},\bar{\boldsymbol{D}}^{-1/2}\right\}\| \\ &\leq \limsup_{n\to\infty}\max\left\{\|\boldsymbol{D}^{1/2}\|,\|\bar{\boldsymbol{D}}^{1/2}\|\right\}\|\boldsymbol{B}-\bar{\boldsymbol{B}}\|\max\left\{\|\boldsymbol{D}^{-1/2}\|,\|\bar{\boldsymbol{D}}^{-1/2}\|\right\} \\ &\leq \limsup_{n\to\infty}\xi\max\left\{\|\boldsymbol{D}^{1/2}\|,\|\bar{\boldsymbol{D}}^{1/2}\|\right\}\max\left\{\|\boldsymbol{D}^{-1/2}\|,\|\bar{\boldsymbol{D}}^{-1/2}\|\right\}\end{aligned}$$

Now since $\xi$ can be chosen to be arbitrarily small, it is sufficient to establish that both $\|\boldsymbol{D}^{1/2}\|_2\|\bar{\boldsymbol{D}}^{-1/2}\|_2$ and $\|\boldsymbol{D}^{1/2}\|_2\|\bar{\boldsymbol{D}}^{-1/2}\|_2$ are bounded by a constant almost surely. To see that they are, observe that

$$\|\boldsymbol{D}^{1/2}\|_2\|\bar{\boldsymbol{D}}^{-1/2}\|_2 = \sqrt{\frac{\max_i\sum_j w_{ij}a_{ij}}{\max_i\sum_j w_{ij}p_{ij}}} \leq \sqrt{\frac{\bar{w}_{(n)}}{\underline{w}_{(n)}}}\max_i\sqrt{\frac{\sum_j a_{ij}}{\sum_j p_{ij}}} \leq \sqrt{\omega}\max_i\sqrt{\frac{\sum_j a_{ij}}{\sum_j p_{ij}}}$$

and similarly

$$\|\boldsymbol{D}^{1/2}\|_2\|\bar{\boldsymbol{D}}^{-1/2}\|_2 = \sqrt{\frac{\max_i\sum_j w_{ij}p_{ij}}{\max_i\sum_j w_{ij}a_{ij}}} \leq \sqrt{\frac{\bar{w}_{(n)}}{\underline{w}_{(n)}}}\max_i\sqrt{\frac{\sum_j p_{ij}}{\sum_j a_{ij}}} \leq \sqrt{\omega}\max_i\sqrt{\frac{\sum_j p_{ij}}{\sum_j a_{ij}}}$$

But since the $a_{ij}$ are distributed $Bernoulli(p_{ij})$, (see e.g. Mostagir and Siderius (2021)) both $\max_i\sqrt{\frac{\sum_j p_{ij}}{\sum_j a_{ij}}}$ and $\max_i\sqrt{\frac{\sum_j a_{ij}}{\sum_j p_{ij}}}$ converge in probability to 1, so that we have for any $\xi > 0$

$$\limsup_{n\to\infty}\|\tilde{\boldsymbol{A}}-\bar{\tilde{\boldsymbol{A}}}\| \leq \xi\sqrt{\omega}$$

That is, the weighted adjacency matrix can be made arbitrarily close to its expected counterpart.

We now wish to show that, for arbitrary $\epsilon > 0$,

$$\lim_{n\to\infty} \Pr(\|(\boldsymbol{I} - \theta\tilde{\boldsymbol{A}})^{-1} - (\boldsymbol{I} - \theta\bar{\tilde{\boldsymbol{A}}})^{-1}\| \geq \epsilon) = 0$$

The key observation is that the above result implies that for any $\mu > 0$, there exists sufficiently large $n$ such that with probability approaching 1, $\|\tilde{\boldsymbol{A}}^k - \bar{\tilde{\boldsymbol{A}}}^k\| \leq \mu$ for all $k$. Then it is straightforward to note that (since by model assumptions we have $\theta < 1$, so the formula for infinite geometric series can be applied),

$$\begin{aligned}
\limsup_{n\to\infty} \|(\boldsymbol{I} - \theta\tilde{\boldsymbol{A}})^{-1} - (\boldsymbol{I} - \theta\bar{\tilde{\boldsymbol{A}}})^{-1}\| &= \limsup_{n\to\infty} \left\| \sum_{k=0}^{\infty} \theta^k \left( \tilde{\boldsymbol{A}}^k - \bar{\tilde{\boldsymbol{A}}}^k \right) \right\| \\
&\leq \limsup_{n\to\infty} \sum_{k=0}^{\infty} |\theta^k| \left\| \left( \tilde{\boldsymbol{A}}^k - \bar{\tilde{\boldsymbol{A}}}^k \right) \right\| \\
&\leq \sum_{k=0}^{\infty} \mu |\theta^k| \\
&= \frac{\mu}{1-\theta}
\end{aligned}$$

Since $\mu$ was chosen arbitrarily, this implies that for any $\epsilon > 0$,

$$\lim_{n\to\infty} \Pr(\|\boldsymbol{c}^{(n)}(\tilde{\boldsymbol{A}}) - \boldsymbol{c}^{(n)}(\bar{\tilde{\boldsymbol{A}}})\| > \epsilon) = 0$$

Finally, note that the assumption of non-vanishing spectral gap guarantees that the network is connected with high probability, so that the centrality is well-defined (Dasaratha 2020; Mostagir and Siderius 2021), completing the proof. □

# References


Alexander, Dan, and Bryan Rooney. 2019. "Vote-Buying by the United States in the United Nations." *International Studies Quarterly* 63(1):168–176.

Auerbach, Adam Michael. 2019. *Demanding development: The politics of public goods provision in India's urban slums.* Cambridge University Press.

Auerbach, Adam Michael, and Tariq Thachil. 2018. "How clients select brokers: competition and choice in India's slums." *American Political Science Review* 112(4):775–791.

Ballester, Coralio, Antoni Calvó-Armengol, and Yves Zenou. 2006. "Who's who in networks. Wanted: The key player." *Econometrica* 74(5):1403–1417.

Banks, Jeffrey S. 2000. "Buying supermajorities in finite legislatures." *American Political Science Review* 94(3):677–681.

Battaglini, Marco, and Eleonora Patacchini. 2018. "Influencing connected legislators." *Journal of Political Economy* 126(6):2277–2322.

Battaglini, Marco, Valerio Leone Sciabolazza, and Eleonora Patacchini. 2020. "Effectiveness of connected legislators." *American Journal of Political Science* 64(4):739–756.

Board, Simon, and Moritz Meyer-ter Vehn. 2021. "Learning dynamics in social networks." *Econometrica* 89(6):2601–2635.

Bonacich, Phillip. 1987. "Power and centrality: A family of measures." *American journal of sociology* 92(5):1170–1182.

Brusco, Valeria, Marcelo Nazareno, and Susan C Stokes. 2004. "Vote buying in Argentina." *Latin American research review* pp. 66–88.

Calvó-Armengol, Antoni, Eleonora Patacchini, and Yves Zenou. 2009. "Peer effects and social networks in education." *The review of economic studies* 76(4):1239–1267.

Calvo, Ernesto, and Maria Victoria Murillo. 2013. "When parties meet voters: Assessing political linkages through partisan networks and distributive expectations in Argentina and Chile." *Comparative Political Studies* 46(7):851–882.

Chandra, Kanchan. 2007. *Why ethnic parties succeed: Patronage and ethnic head counts in India*. Cambridge University Press.

Chung, Fan, and Mary Radcliffe. 2011. "On the spectra of general random graphs." *the electronic journal of combinatorics* pp. P215–P215.

Chung, Fan RK, and Fan Chung Graham. 1997. *Spectral graph theory*. Number 92 American Mathematical Soc.

Cruz, Cesi. 2019. "Social networks and the targeting of vote buying." *Comparative Political Studies* 52(3):382–411.

Cruz, Cesi, Julien Labonne, and Pablo Querubin. 2017. "Politician family networks and electoral outcomes: Evidence from the Philippines." *American Economic Review* 107(10):3006–37.

Dal Bó, Ernesto. 2007. "Bribing voters." *American Journal of Political Science* 51(4):789–803.

Dasaratha, Krishna. 2020. "Distributions of centrality on networks." *Games and Economic Behavior* 122:1–27.

Dekel, Eddie, Matthew O Jackson, and Asher Wolinsky. 2008. "Vote buying: General elections." *Journal of Political Economy* 116(2):351–380.

Drobizheva, Leokadiya. 1999. "Comparison of élite groups in Tatarstan, Sakha, Magadan, and Orenburg." *Post-Soviet Affairs* 15(4):387–406.

Fafchamps, Marcel, and Julien Labonne. 2017. "Do politicians' relatives get better jobs? evidence from municipal elections." *The Journal of Law, Economics, and Organization* 33(2):268–300.

Fearon, James D, and David D Laitin. 1996. "Explaining interethnic cooperation." *American political science review* 90(4):715–735.

Finan, Frederico, and Laura Schechter. 2012. "Vote-buying and reciprocity." *Econometrica* 80(2):863–881.

Frye, Timothy, Ora John Reuter, and David Szakonyi. 2019. "Vote brokers, clientelist appeals, and voter turnout: Evidence from Russia and Venezuela." *World politics* 71(4):710–746.

Gans-Morse, Jordan, Sebastian Mazzuca, and Simeon Nichter. 2014. "Varieties of clientelism: Machine politics during elections." *American Journal of Political Science* 58(2):415–432.

Golosov, Grigorii V. 2011. "The regional roots of electoral authoritarianism in Russia." *Europe-Asia Studies* 63(4):623–639.

Granovetter, Mark S. 1973. "The strength of weak ties." *American journal of sociology* 78(6):1360–1380.

Groseclose, Tim, and James M Snyder. 1996. "Buying supermajorities." *American Political Science Review* 90(2):303–315.

Guardado, Jenny, and Leonard Wantchekon. 2018. "Do electoral handouts affect voting behavior?" *Electoral Studies* 53:139–149.

Gubler, Joshua R, and Joel Sawat Selway. 2012. "Horizontal inequality, crosscutting cleavages, and civil war." *Journal of Conflict Resolution* 56(2):206–232.

Habyarimana, James, Macartan Humphreys, Daniel N Posner, and Jeremy M Weinstein. 2009. *Coethnicity: Diversity and the Dilemmas of Collective Action*. Russell Sage Foundation.

Hale, Henry E. 2003. "Explaining machine politics in Russia's regions: Economy, ethnicity, and legacy." *Post-Soviet Affairs* 19(3):228–263.

Hicken, Allen. 2011. "Clientelism." *Annual review of political science* 14:289–310.

Katz, Leo. 1953. "A new status index derived from sociometric analysis." *Psychometrika* 18(1):39–43.

Keefer, Philip. 2007. "Clientelism, credibility, and the policy choices of young democracies." *American journal of political science* 51(4):804–821.

Keefer, Philip, and Razvan Vlaicu. 2017. "Vote buying and campaign promises." *Journal of Comparative Economics* 45(4):773–792.

Kitschelt, Herbert. 2000. "Linkages between citizens and politicians in democratic polities." *Comparative political studies* 33(6-7):845–879.

Kramon, Eric. 2016*a*. "Electoral handouts as information: Explaining unmonitored vote buying." *World Politics* 68(3):454–498.

Kramon, Eric. 2016*b*. "Where is vote buying effective? Evidence from a list experiment in Kenya." *Electoral Studies* 44:397–408.

Larson, Jennifer M, and Janet I Lewis. 2017. "Ethnic networks." *American Journal of Political Science* 61(2):350–364.

Lizzeri, Alessandro, and Nicola Persico. 2001. "The provision of public goods under alternative electoral incentives." *American Economic Review* 91(1):225–239.

Miguel, Edward, and Mary Kay Gugerty. 2005. "Ethnic diversity, social sanctions, and public goods in Kenya." *Journal of public Economics* 89(11-12):2325–2368.

Mostagir, Mohamed, and James Siderius. 2021. "Centrality in stochastic networks." *Working paper* .

Nichter, Simeon. 2008. "Vote buying or turnout buying? Machine politics and the secret ballot." *American political science review* 102(1):19–31.

Oliveros, Virginia. 2016. "Making it personal: Clientelism, favors, and the personalization of public administration in Argentina." *Comparative Politics* 48(3):373–391.

Putnam, Robert D. 2000. *Bowling alone: The collapse and revival of American community*. Simon and schuster.

Reuter, Ora John, and Graeme B Robertson. 2012. "Subnational appointments in authoritarian regimes: Evidence from Russian gubernatorial appointments." *The Journal of politics* 74(4):1023–1037.

Robinson, James A, and Thierry Verdier. 2013. "The political economy of clientelism." *The Scandinavian Journal of Economics* 115(2):260–291.

Schaffer, Frederic Charles, and Andreas Schedler. 2007. "What is vote buying." *Elections for sale: The causes and consequences of vote buying* pp. 17–30.

Sharafutdinova, Gulnaz. 2013. "Getting the" Dough" and Saving the Machine: Lessons from Tatarstan." *Demokratizatsiya* 21(4).

Stokes, Susan C. 2005. "Perverse accountability: A formal model of machine politics with evidence from Argentina." *American political science review* 99(3):315–325.

Thacker, Strom C. 1999. "The high politics of IMF lending." *World politics* 52(1):38–75.

Vreeland, James Raymond, and Axel Dreher. 2014. *The political economy of the United Nations Security Council: Money and influence*. Cambridge University Press.

White, Allison C, and Inga A-L Saikkonen. 2017. "More than a name? Variation in electoral mobilisation of titular and non-titular ethnic minorities in Russian national elections." *Ethnopolitics* 16(5):450–470.